Dr. Philipp Wruck 27.10.2011

Exercise Sheet 1
Transformation Groups

Exercise 1: Let G_i be topological groups for i in some index set I. Show that

$$\prod_{i \in I} G_i$$

is a topological group with componentwise multiplication and the product topology.

Exercise 2:

(i) Let $\mathcal{HAUS}$ be the category of Hausdorff spaces and continuous maps. Let Y be a Hausdorff space, let $\bullet \times Y : \mathcal{HAUS} \to \mathcal{HAUS},\ X \mapsto X \times Y$ be the functor sending $f : X \to Z$ to the map $f \times \mathrm{id}_Y : X \times Y \to Z \times Y,\ (x, y) \mapsto (f(x), y)$ and let $\mathcal{HAUS}(Y, \bullet) : \mathcal{HAUS} \to \mathcal{HAUS},\ Z \mapsto \mathcal{HAUS}(Y, Z)$ be the functor sending $f : X \to Z$ to the map $\mathcal{HAUS}(Y, f) : \mathcal{HAUS}(Y, X) \to \mathcal{HAUS}(Y, Z),\ g \mapsto f \circ g$. Show that if Y is locally compact, then $\bullet \times Y$ is left adjoint to $\mathcal{HAUS}(Y, \bullet)$. Here and in the following, function spaces carry the compact-open topology.

(ii) Under the assumptions of (i), prove that evaluation

$$\mathcal{HAUS}(X, Y) \times X \to Y,\ (f, x) \mapsto f(x)$$

is continuous.

(iii) Assume in addition to the assumptions of (i) that X is locally compact as well. Prove that composition

$$\mathcal{HAUS}(Y, Z) \times \mathcal{HAUS}(X, Y) \to \mathcal{HAUS}(X, Z),\ (f, g) \mapsto f \circ g$$

is continuous.

Exercise 3: Let X be a compact Hausdorff space. Prove that $Homeo(X)$ with the compact-open topology is a topological group.

Handing in: 10.11.11 in exercise class.

Action of the Week

Group:	$O(n)$
Space:	$\mathbb{S}^{n-1}$
Action:	$(A, x) \mapsto A \cdot x$
Isotropies:	$O(n-1)$
Further properties:	transitive, monotypic