Exercises for Higher Structures in Differential Geometry SS 2013

Sheet 07

Exercise 34

Show inductively that if $U \Subset X$ and $V \Subset Y$, then

$$d^k f(x)(v_1, ..., v_k) = \mathrm{pr}_{2^k}\left(T^k(x, w_1, ..., w_{2^k-1})\right)$$

with $w_{2^i+1} = v_{i+1}$ for $0 \leq i \leq k-1$ and $w_i = 0$ else.

Exercise 35

Show that if X, Y, Z are Hausdorff spaces and $f\colon X \times Y \to Z$ is continuous, then for each $x \in X$ we have $\widehat{f}(x) \in C(Y, Z)$ and $\widehat{f}\colon X \to C(Y, Z)_{c.o.}$ is continuous.

Exercise 36

Show that if X, Y are topological spaces and X is Hausdorff and locally compact (i.e., each point in X has a compact neighbourhood), then the evaluation map

$$\mathrm{ev}\colon C(X, Y) \times X \to Y, \quad (\gamma, x) \mapsto \gamma(x)$$

is continuous.

Exercise 37

Show that if M, N are manifolds, M is compact, N is locally metrisable and $O \subseteq N$ is open, then $C^\infty(M, O)$ is open in the d-topology on $C^\infty(M, N)$.

Exercise 38

Show that for a Hausdorff space X and a metrisable space Y the topology of compact convergence equals the compact open topology. **Hint:** This involves various typical compactness arguments.