Exercises for Algebra II, WS 12/13

Sheet 8 – Solutions

Exercise 35

a) This is clear, the category of additive categories with only one object is isomorphic to the category of rings.

b) One can define the additive structure point-wise.

c) Each functor assigns to the single object one abelian group M and to each morphism of $\mathcal{C}$ (each element $r \in R$) an additive map $M \to M$, which we may identify with the scalar multiplication by r. This describes actually an isomorphism of categories.

Exercise 36

Let $f_\bullet \colon A_\bullet \to B_\bullet$ a morphism of chain complexes. If $g_\bullet \colon C_\bullet \to A_\bullet$ is another morphism such that $f_\bullet \circ g_\bullet = 0$, then $g_\bullet(C_\bullet) \subseteq \ker(f_\bullet)$. Thus there exists a unique morphism $g_\bullet \colon C_\bullet \to \ker(f_\bullet)$ making the respective diagram commute (simply restrict the target to $\ker(f_\bullet)$). Thus $\ker(f_\bullet)$ is a kernel of $f_\bullet$. The same argument also works for $\operatorname{coker}(f_\bullet)$.

Exercise 37

a) $\mathbf{Ab}^{\text{fin}}$ is a subcategory of $\mathbf{Ab}$. Since the usual kernels and cokernels of morphisms between finite abelian groups are again finite, the corresponding requirements for $\mathbf{Ab} = \mathbb{Z}\textbf{-Mod}$ show that $\mathbf{Ab}^{\text{fin}}$ is abelian.

b) The same argument as the one in Ex. 36 shows that in **Ch(R-Mod)** the additive additive structure, kernels and cokernels are simply defined degree-wise. Thus the compatibility of kernels and cokernels for mono- and epimorphisms may also be checked degree-wise. This shows that **Ch(R-Mod)** is also abelian.

Exercise 38

The proof is spelled out in the lecture notes of Chr. Schweigert (Lemma 1.5.8).