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Winter Term 2019/20

Optimization of Complex Systems – 10th Exercise Sheet.

Discussion of the solutions in the exercise on January 27, 2020.

Problem 1 (primal dual active set strategy): Consider the optimal control problem from HW 9/3 and the discretization from sub-problem a). In the primal dual active set method, in each step a linear system of equations of the form

$$\begin{bmatrix} 0 & A & -R \\ A & -M & 0 \\ ER^{\mathsf{T}} & 0 & I \end{bmatrix} \begin{bmatrix} \widehat{p} \\ \widehat{y} \\ \widehat{u} \end{bmatrix} = \begin{bmatrix} 0 \\ -M\widehat{y}_{\mathrm{d}} \\ X_n^a \widehat{u}_a + X_n^b \widehat{u}_b \end{bmatrix}, \tag{1}$$

is solved, where X_n^a and X_n^b are diagonal matrices with

$$X_{n,ii}^a = \begin{cases} 1 & \text{if } i \in A_n^a, \\ 0 & \text{otherwise} \end{cases}, \quad X_{n,ii}^b = \begin{cases} 1 & \text{if } i \in A_n^b, \\ 0 & \text{otherwise} \end{cases},$$

and

$$E := (\lambda N)^{-1}\big(I - X_n^a - X_n^b\big).$$

a) Show that the linear system (1) is equivalent to a *symmetric* linear system

$$\begin{bmatrix} 0 & A & RE_1 \\ A & -M & 0 \\ E_1^{\mathsf{T}} R^{\mathsf{T}} & 0 & -E_2 \end{bmatrix} \begin{bmatrix} \widehat{p} \\ \widehat{y} \\ \widetilde{u} \end{bmatrix} = \begin{bmatrix} 0 \\ -M\widehat{y}_{\mathrm{d}} \\ 0 \end{bmatrix},$$

for some matrices E_1, E_2 of appropriate dimension and where the length of $\widetilde{u}$ corresponds to the number of inactive indices. Specify E_1 and E_2.

b) Implement the primal dual active set strategy in MATLAB and test it for various levels of discretization on the example from HW 9/3. Visualize your results, i. e., plot the optimal control and corresponding optimal state function.

Problem 2 (saddle point matrices): Consider the matrix

$$M = \begin{bmatrix} A & B \\ B^{\mathsf{T}} & 0 \end{bmatrix}$$

with symmetric and negative definite $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ with $\operatorname{rank} B = m$. Show that M has exactly m positive and n negative eigenvalues.