Complex Geometry
WS 16/17

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Exercises 8

1. Let $\varphi : \mathcal{F} \to \mathcal{G}$ be a morphism of abelian sheaves on the topological space X. Show:

(a) For any $p \in X$ it holds $(\ker \varphi)_p = \ker \varphi_p$.
(b) For any $p \in X$ it holds $(\operatorname{im} \varphi)_p = \operatorname{im} \varphi_p$.

2. For a presheaf $\mathcal{F}$ on a topological space X define its *étale space* in the following way. As a set

$$\text{Ét}(\mathcal{F}) = \coprod_{p \in X} \mathcal{F}_p.$$

Any section $s \in \mathcal{F}(U)$ with $U \subset X$ open defines a map

$$\sigma_s : U \longrightarrow \text{Ét}(\mathcal{F}), \quad p \longmapsto s_p.$$

Endow $\text{Ét}(\mathcal{F})$ with the following topology: $V \subset \text{Ét}(\mathcal{F})$ is open iff for any $U \subset X$ open and $s \in \mathcal{F}(U)$, the preimage $\sigma_s^{-1}(V) \subset U$ is open. Denote by

$$\pi : \text{Ét}(\mathcal{F}) \to X$$

the projection map sending $\mathcal{F}_p$ to p.

(a) Show that π is continuous and that the sheaf of continuous sections of $\pi : \text{Ét}(\mathcal{F}) \to X$ is canonically isomorphic to the sheaf $\mathcal{F}^+$ associated to the presheaf $\mathcal{F}$.

(b) If X is a complex manifold then the étale space of $\mathcal{O}_X$ is Hausdorff, while the étale space of the sheaf $\mathcal{C}^0$ of continuous functions is not unless $\dim X = 0$.

(c) Let A be an abelian group. A sheaf $\mathcal{F}$ on a topological space X is called a *local system with fibres A* if it is locally isomorphic to the constant sheaf with fibres A. Show that for a local system, $\pi : \text{Ét}(\mathcal{F}) \to X$ is a topological covering map.