Complex Geometry Prof. Dr. Bernd Siebert
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Exercises 3

1. Let $U \subset \mathbb{C}^n$ be open. A *meromorphic function* on U is a holomorphic function on $U \setminus S$ for some nowhere dense closed subset $S \subset U$, which locally in U is given by a quotient of holomorphic functions. Meromorphic functions are considered equivalent if they agree outside some common nowhere dense closed subset.

Show that the ring $K(U)$ of meromorphic functions on U is a field if and only if U is connected. [Hu, Ex.1.1.9]

(Reminder: A topological space X is called *connected* if any decomposition $X = U \cup V$ in disjoint open subsets is trivial, that is, $U = X$ or $V = X$.)

2. Sketch the proof of factoriality of the polynomial ring $R[x]$ over a factorial ring R following a book on algebra (e.g. Serge Lang's "Algebra", Ch.IV §2).