

Exercises 12

1. For $A \in \mathrm{GL}(n+1, \mathbb{C})$ denote by $F_A : \mathbb{P}^n \rightarrow \mathbb{P}^n$ the induced biholomorphism.

Show: $F_A^*(\omega_{\mathrm{FS}}) = \omega_{\mathrm{FS}} \iff A \in U(n+1)$.

2. Show: $\int_{\mathbb{P}^n} \omega_{\mathrm{FS}} = 1$.

3. Let X be a compact Kähler manifold of dimension n . Show that $x \mapsto [\omega]$ defines an injective ring homomorphism

$$\mathbb{R}[x]/(x^{n+1}) \longrightarrow H_{dR}^*(X).$$

Hint: $\int_X \omega^{\wedge n} > 0$.

4. Show that the product of two Kähler manifolds is a Kähler manifold.