

Exercises 1

1. Compute

$$\partial_{\bar{z}} \left(\frac{e^{|z|^3}}{(\bar{z} - z)^2} \right), \quad \partial_z^2 \partial_{\bar{w}} ((w - \bar{z}^3)(\bar{w}^3 - z)).$$

2. Write the Laplace operator

$$\Delta = \sum_{i=1}^n \partial_{x_i}^2 + \sum_{i=1}^n \partial_{y_i}^2$$

on \mathbb{C}^n in terms of $\partial_{z_i}, \partial_{\bar{z}_i}$.

3. State and prove maximum principle and identity theorem for holomorphic functions in several variables from the corresponding result in one variable [Hu, Ex. 1.1.3].

4.* Let $A \subset \mathbb{C}^2$ be the zero set of the Weierstraß polynomial

$$f(z, w) = w^3 + z \cdot w^2 - 4e^z.$$

Determine the set $\{z \in \mathbb{C} \mid |(\{z\} \times \mathbb{C}) \cap A| < 3\}$.

5. Construct a holomorphic function f on the unit disk that does not extend to any larger domain.