SUSY-QFT and VOA's

– Motivation and Overview –

QFT: Predicts average values of quantum field measurements around points, e.g

$$\int d^{n-1}\vec{x} \int dt \, \langle \vec{E}(\vec{x},t) \rangle \, \rho(\vec{x},t)$$

Observable / operator assoc. to electric field

Abstractly: QFT $\sim$ $(\mathcal{A}, \mathcal{H}, P_\mu, \dots)$

alg. of ops

$\mathcal{O} = \int d^n x \, \varphi(x) \, f(x)$

rep. of $\mathcal{A}$

rep. of transl. on $\mathcal{H}$ *)

*) $P_\mu \varphi(x) = \partial_\mu \varphi(x)$,

SUSY: Extension of Translation group (graded / super Lie alg)

Example: 3d $(n=3)$, $\mathcal{N}=2$ SUSY

Costello
Dimofte
Gaiotto

Generators $Q_{+,-}$, $\bar{Q}_\pm$, relations

$$\{Q_+, \bar{Q}_+\} = -2i\, D_{\bar{z}} \quad , \quad \{Q_-, \bar{Q}_-\} = 2i\, D_z$$

$$\{Q_+, \bar{Q}_-\} = \{Q_-, \bar{Q}_+\} = i\partial_t \, ,$$

using $\mathbb{R}^3 \simeq \mathbb{C} \times \mathbb{R}$

cplx. coord $z, \bar{z}$

real coord. t

and representation on $(\mathcal{A}, \mathcal{H})$.

Note: $Q := \bar{Q}_+$ nilpotent, $Q^2 = 0$

Twisting:

Restrict attention to cohomology of Q:

$$\mathcal{H}_Q := \{ \psi \in \mathcal{H} \,;\, Q\psi = 0 \} \big/ Q\mathcal{H}$$

$$\mathcal{V} = \{ \mathcal{O} \in A \,;\, [Q, \mathcal{O}] = 0 \} \big/ [Q, A]$$

Important: $(\mathcal{V}, \mathcal{H}_Q)$ can capture crucial info, (physics at low energies)

$$i\partial_t \langle \varphi(x) \rangle \quad \langle [\{ Q_-, \bar{Q}_+ \}, \varphi(x)] \rangle$$

$$= \underbrace{\langle \{ [Q_-, \varphi(x)], \bar{Q}_+ \} \rangle}_{Q\text{-inv. of } \langle \ldots \rangle} + \langle \{ \overset{Q}{\overset{\parallel}{Q_-}}, \underbrace{[\bar{Q}_+, \varphi]}_{0 \ (\varphi \in \mathcal{V})} \} \rangle$$

$$= 0$$

$i\partial_{\bar{z}} \langle \varphi(x) \rangle = 0$ — similar proof

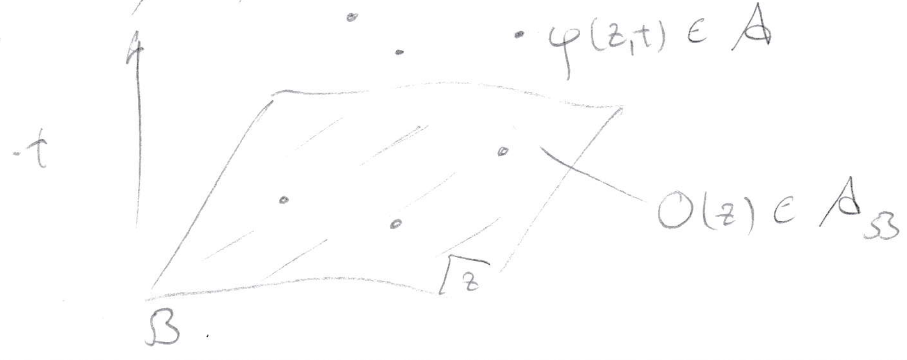
$\Longrightarrow$ φ holomorphic, $\varphi(x) \equiv \varphi(z)$.

It follows that $\langle \varphi(z,t)\, \varphi'(z',t') \ldots \rangle$ is <u>not</u> singular for $z = z'$, $t = t'$!

$\rightsquigarrow \ldots \rightsquigarrow$ $\mathcal{V}$: <u>commutative</u> Poisson-VOA

$\mathcal{B}$

Enrich story by considering defects/boundaries



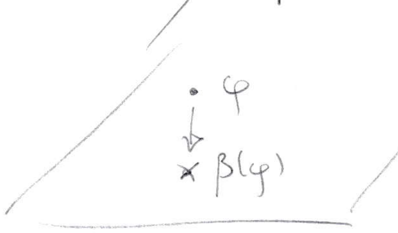
Consider $\mathcal{V}_{\mathcal{B}} = \{ \mathcal{O} \in A_{\mathcal{B}} \mid [Q, \mathcal{O}] = 0 \} / [Q, A_{\mathcal{B}}]$

As before $\partial_{\bar{z}} \langle \mathcal{O}(z) \rangle = 0 \quad \forall \mathcal{O} \in \mathcal{V}_{\mathcal{B}}$.

But $\langle \mathcal{O}(z) \mathcal{O}'(z') \ldots \rangle$ can be singular at $z = z'$

$\rightsquigarrow$ VOA of holomorphic boundary ops.

Bulk-boundary map β:



$$\beta : \mathcal{V} \hookrightarrow \underbrace{Z(\mathcal{V}_{\mathcal{B}})}_{} \hookrightarrow \mathcal{V}_{\mathcal{B}}$$

$\mathcal{O} \in \mathcal{V}_{\mathcal{B}}$ s.t. $\mathcal{O}(z)\mathcal{O}'(z')$ non-sing $\forall \mathcal{O}'$

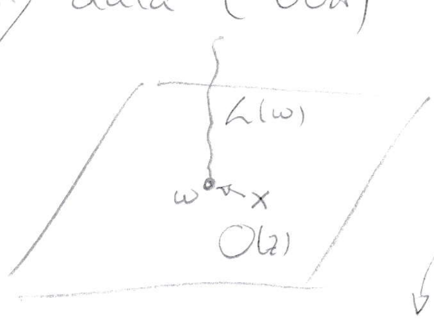
In good cases: β isomorphism. May then

|| Characterize bulk-alg in terms of bd-VOA ||

Motiv. phys.

Reconstruct 3d vacuum physics ($\mathcal{V}$ a.k.a. chiral ring) from boundary data (VOA)

Line operators:



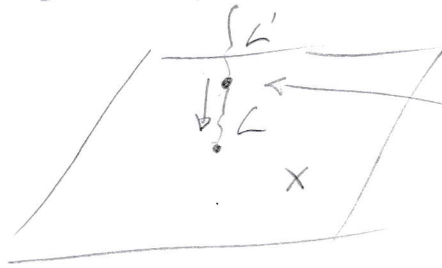
M_L

OPE $\mathcal{O}(z) L(w) \rightsquigarrow L(w)$ - module for VOA

Resulting picture resembles holographic relation 2d-CFT to 3d CS

3d TQFT on $M^3 \rightsquigarrow$ 2d CFT on ∂M^3

.... but is much richer:



$\in \mathrm{Hom}_{\mathcal{V}_{S3}}(L, L')$

especially

(trivial line)

$\simeq \mathcal{V}_{S3}$

$\in \mathrm{Ext}(M_{\mathbb{1}}, M_{\mathbb{1}})$

These objects are non-trivial iff category of $\mathcal{V}_{S3}$-rep non-semisimple!

Analogous (related?!): 4d - VOA - corresp.:

Observation (Beem, Lemos, Rastelli ...):

$\exists$ generator Q of $su(2,2|2)$, $Q^2 = 0$

s.t. $\mathcal{V}_Q := \{ \mathcal{O} \in \mathcal{A} \mid [Q, \mathcal{O}] = 0 \} / [Q, \mathcal{A}]$

- only contains operators supported on $\mathbb{C} \simeq \mathbb{R}^2 \subset \mathbb{R}^4$
- elements of $\mathcal{V}_Q$ have OPEs holomorphic w.r.t. coord. z for $\mathbb{C} \subset \mathbb{R}^4$.

$\rightsquigarrow$ another VOA

Algebraic reconstruction of Higgs branch:

$$\mathcal{R}_H \simeq X_{\mathcal{V}_Q}$$

Higgs branch chiral ring

associated variety of VOA $\mathcal{V}_Q$ (Arakawa)