

3d SUSY QFT

$$\overline{S}_{G,k}^A = (TLGJ/G_p)^A$$

Pair of dual VOA $\tilde{N}_{G,k}, D_{G,k}$

VOAs

• $G^\vee$ global symm. G gauge symm.

• $\exists$ Lagrangian (BV)

Key data:

(i) Algebras U, U_2 of bulk/bd. ops.

(ii) Spaces of states $\mathcal{H}(\Sigma)$

(iii) Category $\mathcal{E}$ of line ops

Category $\mathcal{E}$ of line-ops.
Repr. $\simeq$ category of VOA

A-B

(i) $\text{Ext}(\text{Vac}, \text{Vac})$

(ii) (Derived) spaces of conformal blocks

(iii) Representation categories (braided $\otimes$ -cat.)

Lagrangian descriptions
+ 3d mirror symm.

A
D

A	B
D	C

B
C

Kazhdan-Lusztig
type equivalences
of braided $\otimes$ -cat.

Nilpotent cone $N^\vee$, Springer resolution

(i) Ring of functions $\mathbb{C}[T^*[2]\mathbb{S}^\vee]$

(ii) $H^*(\text{Bun}_{G_e}(\Sigma), \mathcal{E}_{TLGJ^A}(\Sigma)|_A \otimes \mathcal{L}^\vee)$

(iii) "A version of D-mod($V(\mathbb{C}[z])/G(\mathbb{C}[z])$)"

Geometric
representation
theory

D-C

(Small) quantum group $U_q(\mathfrak{g})$ ($u_q(\mathfrak{g})$)

(i) $\text{End}_{\mathcal{E}}(1)$, 1 : id-object

(ii) $\mathcal{H}_{\text{TQFT}}(\Sigma)$, esp.

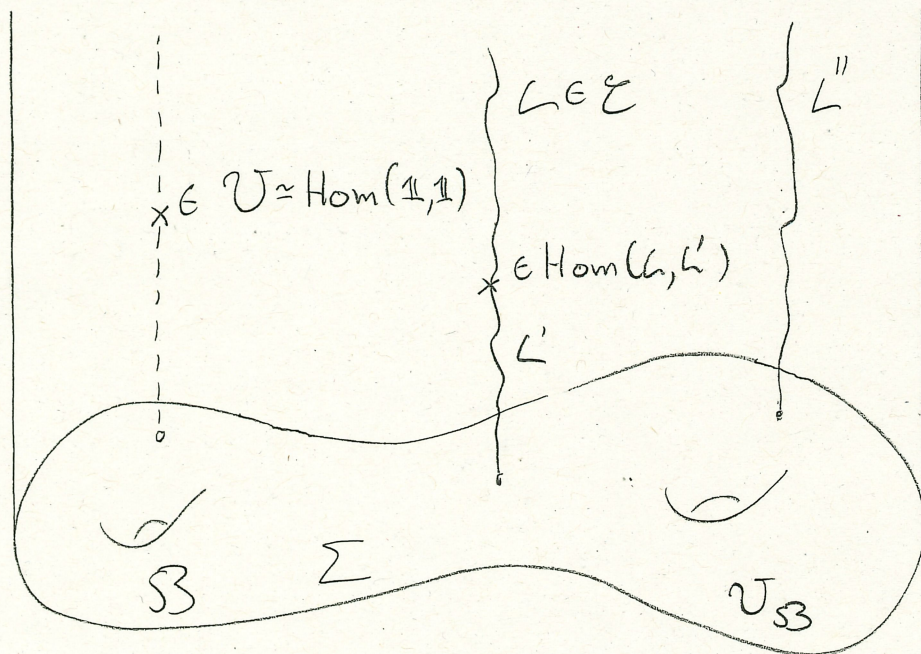
$$\mathcal{H}_{\text{TQFT}}(T^2, A=0) = \text{HH}_*(\mathcal{E}_1)$$

(iii) Representation
category $\mathcal{E}$

Geometry

TQFT

Quantum-group



S_B : boundary cond (N/D)

$$\mathcal{N} = \{ a \in \mathfrak{g}; \quad g(a) \text{ nilpotent in all reps.} \\ g: \mathfrak{g} \rightarrow \text{End}(V) \}$$

Springer resolution of $\mathcal{N} \simeq T^*[2]\overline{\mathfrak{g}}^\vee$

$$\Sigma_{T[\mathfrak{g}]^A}(S^2) \simeq \mathbb{C}[T^*[2]\overline{\mathfrak{g}}^\vee]$$

$$\Sigma_{T[\mathfrak{g}]^A}(T^2) \simeq H_{\overline{2}}^{\bullet}(T^*[2]\overline{\mathfrak{g}}^\vee)$$

A	B
D	C

$$\tilde{\mathcal{N}}_{G,k} \simeq \text{Com}(\bigvee^k(\mathfrak{sl}_n), A(\mathfrak{sl}_n) \otimes \text{FF})$$

Built from coset / extension of
affine VOA, lattice VOA, and FF

$\mathcal{D}_{G,k}$: Extension of W-alg $W^{1/k}(\mathfrak{g})$

$$\mathcal{D}_{G,k} = \bigoplus_{\lambda \in Q^+} R_{\lambda} \otimes W_{\lambda,0}^{1/k}$$

Feigin-Tipunin-alg

Duality $\tilde{\mathcal{N}}_{G,2} \simeq \mathcal{D}_{G,k}$: commutants in FF

Quantum group

Gens. $\{E_i, F_i, K_i^{\pm 1}\}_{i=1}^{\text{rk } \mathfrak{g}}$ + relations, $q = e^{\frac{2\pi i}{r}}$

Center: Gens. $E_i^\Gamma, F_i^\Gamma, K_i^{\pm 2r} \sim \text{fct. on } G_c^\vee$

Rep.-Cat $\mathcal{E} = \bigoplus_{g \in G_c^\vee} \mathcal{E}_g$ — "blocks"

(sheaf of categories over $G_c^\vee$)

Stalk at $g=1$: Reps. of "small" g -grp.

$$E_i^\Gamma = 0, \quad F_i^\Gamma = 0, \quad K_i^{\pm 2r} = 1.$$

a) Computation of U_2 from Lagrangian of $\mathcal{T}_{G,2}^A$

b) Relation to $N=4$ SYM and "corner VOA"

a) Costello - (Creutzig / Dimofte) - Gaiotto

b) Gaiotto, Creutzig - Gaiotto
Gaiotto - Rapčák

Kazhdan - Lusztig
equivalences:

Equivalences of braided $\otimes$ -cat

VOA-mod $\simeq$ (q-group)-mod

Important special cases:

Creutzig - Lentner - Rupert

Gannon - Neynon

Coulomb branch M_{Coul}

$\simeq$ nilpotent cone $\mathcal{N}^\vee$

$\xrightarrow{\text{mass def.}} M_{\text{Coul}} \simeq T^*[S^\vee]$

Springer resolution, $\overline{S}^\vee = G_c^\vee / B_c^\vee$

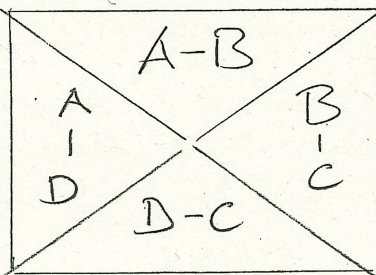
Checked using Lagrangian QFT

(localization ...) and

3d mirror symmetry

Bulimore et al, Gukov, et al

Kapustin et al.



Geometric representation
theory

$$H_2^{\text{ss}}(T^*[S^\vee]) \simeq HH_0(\mathcal{E})$$

(Hochschild homology)

Arkhipov - Bezrukavnikov - Ginzburg

Bezrukavnikov - Lachowska; Lachowska - Qi