Spaces and Differential Forms

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Abstract

We propose a setup of concepts that is supposed to neatly capture the notions of smooth spaces, Lie ∞-groupoids and Lie ∞-algebras and the relations between these.

Contents

1 Introduction

We are after a general framework and tool set for smooth analysis neatly adapted to encompassing

$$\begin{array}{ccc} & \text{smooth spaces} & \\ \swarrow\!\!\nearrow & & \nwarrow\!\!\searrow \\ \text{Lie } \infty\text{-groupoids} & \longleftrightarrow & \text{Lie } \infty\text{-algebras} \end{array}$$

and suited for the description of quantum field theories of Σ-model type: representations of cobordism categories induced from homs into smooth ∞-bundles with connection.

The following approach has its roots in, and is hoped to eventually be a useful synthesis of,

- the emerging Lie ∞-theory of [4, 5, 11];
- the notion of Chen-smooth [6] and diffeological spaces [3] and in particular of *Frölicher* spaces [13];
- the notion of C^∞-algebras [8];
- long discussion with John Baez, Andrew Stacey and Todd Trimble [9, 13, 14].

As Todd Trimble points out, various of the following constructions are special cases of general concepts that Lawvere has taught are important [7].

$$\text{Lawvere:} \qquad \text{space} \xleftrightarrow{\text{conjugation}} \text{quantity}$$

$$\text{presheaf} \xleftrightarrow{\mathrm{Hom}(-,-)} \text{co-presheaf}$$

$$\text{here:} \qquad \begin{array}{c}\text{smooth}\\ \text{space}\end{array} \longleftrightarrow \begin{array}{c}\text{smooth}\\ \text{functions}\end{array}$$

$$\text{Lawvere:} \qquad \text{space} \xleftrightarrow{\begin{array}{c}\text{ambimorphic}\\ \text{object}\end{array}} \begin{array}{c}\text{dual}\\ \text{object}\end{array}$$

$$\text{here:} \qquad \text{space} \xleftrightarrow{\Omega^\bullet(-)} \begin{array}{c}\text{differential}\\ \text{forms}\end{array}$$

As emphasized by Andrew Stacey, of particular importance are those spaces, which are *stable* under conjugating back and forth. Here we will identify such stable spaces as *smooth spaces*, generalizing the notion of *Frölicher spaces*.

2 Space and quantity

In [7] Lawvere describes the very general setup of which we want to consider a special realization here.

For V any monoidal category and S any V-enriched category (a category whose Hom-things are objects of V) the category

$$V^{S^{\mathrm{op}}}$$

of V-functors $S^{\mathrm{op}} \to V$ plays the role of *spaces that can be probed by* A while the category

$$V^S$$

of V-functors $S \to V$ plays the role of *quantities on these spaces*.

We will concentrate on the familiar simple case where $V = \mathrm{Set}$, so that V-enriched categories are just ordinary categories. In this case $\mathrm{Set}^{S^{\mathrm{op}}}$ is just the category of ordinary presheaves on S, while Set^S is the category of ordinary co-presheaves on S.

Definition 1 (Isbell conjugation) *Isbell conjugation is the contravariant adjunction*

$$V^{S^{\mathrm{op}}} \underset{G}{\overset{F}{\rightleftarrows}} V^S$$

given by

$$F : X \mapsto \mathrm{Hom}_{V^{S^{\mathrm{op}}}}(X, -)$$

and

$$G : S \mapsto \mathrm{Hom}_{V^S}(S, -)\,.$$

Here we are, for convenience, implicitly using the Yoneda embedding in order to regard objects $s \in S$ as objects $\mathrm{Hom}_S(-, s) \in \mathrm{Set}^{S^{\mathrm{op}}}$ or objects $\mathrm{Hom}_S(s, -) \in \mathrm{Set}^S$.

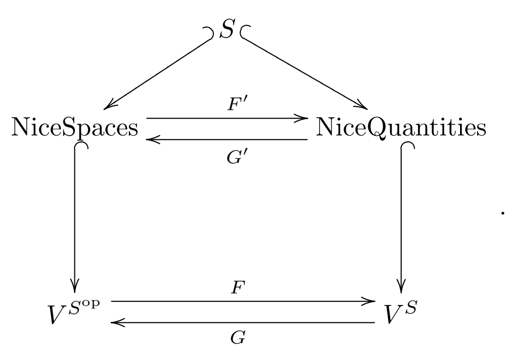
We can think of F as *sending a space to the collection of functions on it.*

The notion of a *space probeable by* S expressed by $V^{S^{\mathrm{op}}}$ is very general. Usually one is therefore interested in finding subcategories

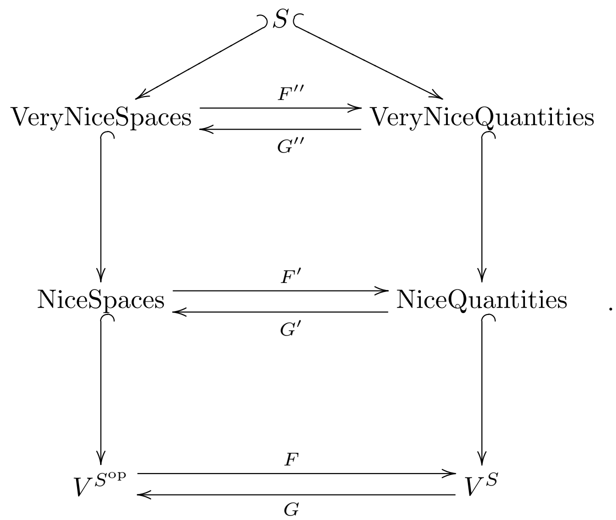
$$S \hookrightarrow \mathrm{NiceSpaces} \hookrightarrow V^{S^{\mathrm{op}}}$$

$$S \hookrightarrow \mathrm{NiceQuantities} \hookrightarrow V^{S}$$

which still respect the above conjugation in that we have



Often one wants to consider chains of such inclusions



In our application we take S to be the category whose objects are the simplest objects we may want to probe a general *smooth space* with: open subsets of Euclidean spaces.

A presheaf on S is a very general notion of a smooth space.

Our "nice spaces" will be proper *sheaves* on S. Our "very nice spaces" will be sheaves on S which are stable under Isbell conjugation: a morphisms of those is the same as a morphism of their function algebras.

One can think of Isbell conjugation as a special case of a general "duality" operation induced by "ambimorphic objects" Amb (originally called "schizophrenic objects" [12]) which can be regarded as carrying two different "commuting" structures. For Isbell conjugation this ambimorphic object is the tautological one,

$$C^\infty(-) = \mathrm{Hom}(-,-)\,,$$

regarded as a co-presheaf valued presheaf.

It so happens that Lie theory is closely related to *differential* algebras (at the bottom of this phenomenon is another grand duality: Koszul duality for operads) and therefore we will wish to refine the algebra $C^\infty(X)$ of plain functions on a space X by the differential $\mathbb{N}$-graded-commutative algebra (DGCA) of differential forms $\Omega^\bullet(X)$.

The presheaf

$$\Omega^\bullet(-) : S^{\mathrm{op}} \to \mathrm{Set}$$

which sends each test domain to the set

$$U \mapsto \Omega^\bullet(X)$$

of differential forms on it is naturally equipped with the structure of a DGCA itself, induced from the the DGCA structure on each test domain. The DGCA-valued presheaf $\Omega^\bullet$ is an ambimorphic object and the two functors

$$\Omega^\bullet : \mathrm{Set}^{S^{\mathrm{op}}} \to \mathrm{DGCAs}$$

$$X \mapsto \mathrm{Hom}_{\mathrm{Set}^{\mathrm{op}}}(X, \Omega^\bullet(-))$$

and

$$S : \mathrm{DGCAs} \to \mathrm{Set}^{S^{\mathrm{op}}}$$

$$A \mapsto \mathrm{Hom}_{\mathrm{DGCAs}}(A, \Omega^\bullet(-)))$$

it induces do form an adjunction

$$\mathrm{Set}^{S^{\mathrm{op}}} \underset{S}{\overset{\Omega^\bullet}{\rightleftarrows}} \mathrm{DGCAs}\ .$$

3 Smooth spaces and smooth differential forms

The following long definition lists the collection of concepts which we want to use. It essentially amounts to fixing a category S of suitable "test domains" and identifying various categories of maps into and out of S as usefully representing spaces and functions on them.

Our choice of S is mostly motivated from its convenience for the particular applications we are headed to. Various other choices should be possible with only minor effect on the resulting theory.

Definition 2 ((smooth) spaces and (smooth) function algebras) *We write*

- S *for the* **category of open subsets of Euclidean spaces**, *whose objects are open subsets of* $\coprod_{n\in\mathbb{N}} \mathbb{R}^n$ *and whose morphisms are* smooth *maps between these;*
  - $S' \subset S$ *for the full* **subcategory of Euclidean spaces** *on the objects* $\mathbb{R}^n$, $n \in \mathbb{N}$, *which we shall always regard as a* symmetric monoidal *category* $(S', \otimes)$ *using the standard cartesian product* $\mathbb{R}^n \otimes \mathbb{R}^m = \mathbb{R}^{n+m}$*;*
- $C^\infty\mathrm{Algebras}$ *for the* **smooth commutative algebras** *being the category of* monoidal *functors* $S' \to \mathrm{Set}$*;*
  - $\mathrm{ev}_{\mathbb{R}} : C^\infty\mathrm{Algebras} \to \mathrm{CommAlgebras}$ *for the functor* $\mathrm{ev}_{\mathbb{R}} : A \mapsto A(\mathbb{R})$ *which sends each* C^∞*-algebra* A *to its* **underlying commutative algebra** $A(\mathbb{R})$ *which comes naturally equipped with the structure of an ordinary commutative algebra, the product being* $A(\ \mathbb{R}\times\mathbb{R} \xrightarrow{\cdot} \mathbb{R}\ )$*;*
  - $C^\infty(U) := \mathrm{Hom}_S(U, -) : S' \to \mathrm{Set}$ *for the smooth algebra of* **smooth functions** *on* $U \in S$*;*
- Spaces *for the* **category of spaces** *"probeable by* S*", defined to be the category of sheaves on* S*;*
  - $X \times Y : U \mapsto X(U) \times Y(U)$ *for the cartesian* **product of spaces** $X, Y \in \mathrm{Spaces}$*;*
  - $\mathrm{hom}(X, Y) : U \mapsto \mathrm{Hom}_{\mathrm{Spaces}}(X \times U, Y)$ *for* **space of maps of spaces** $X, Y \in \mathrm{Spaces}$ *(the internal hom of* Spaces*);*
  - $C^\infty(X) := \mathrm{Hom}_{\mathrm{Spaces}}(X, -) : S' \to \mathrm{Set}$ *for the* **smooth algebra of smooth functions** *on* $X \in \mathrm{Spaces}$*;*
  - $C^\infty\mathrm{Spaces}$ *for the category of* **smooth spaces**, *being the full subcategory of* Spaces *on* saturated *or* Frölicher *spaces, which are those spaces* X *satisfying*

$$X \simeq \mathrm{Hom}_{C^\infty\mathrm{Algebras}}(C^\infty(X), C^\infty(-))\,.$$

- $\Omega^\bullet : S \to \mathrm{DGCAs}$ *for the* DGCA*-valued sheaf of* **differential forms***;*
  - $\Omega^\bullet(X) := \mathrm{Hom}_{\mathrm{Spaces}}(X, \Omega^\bullet)$ *for the* **DGCA of differential forms** *on* $X \in \mathrm{Spaces}$*; or* $\Omega^\bullet(X) := \mathrm{hom}_{\mathrm{Spaces}}(X, \Omega^\bullet)$ *if we want to regard* $\Omega^\bullet(X)$ *itself as a space;*
  - $S(A) := \mathrm{Hom}_{C^\infty\mathrm{Algebras}}(A, \Omega^\bullet(-))$ *for the space obtained by regarding* $A \in C^\infty\mathrm{Algebras}$ *as a DGCA of differential forms.*

3.1 Examples

3.1.1 Chen-smooth spaces

Definition 3 *A space X is a* Chen space *[13] or a* quasi-representable space *if there exists a set X_s such that*

$$\begin{array}{ccc} X(U) & \hookrightarrow & \mathrm{Hom}_{\mathrm{Set}}(U, X_s) \\ \uparrow{\scriptstyle X(\phi)} & & \uparrow{\scriptstyle \mathrm{Hom}_{\mathrm{Set}}(\phi, X_s)} \\ X(V) & \hookrightarrow & \mathrm{Hom}_{\mathrm{Set}}(V, X_s) \end{array}$$

for all morphisms ($U \xrightarrow{\phi} V$) *in S, where the inclusions*

$$X(U) \hookrightarrow \mathrm{Hom}_{\mathrm{Set}}(U, X_s)$$

are required to contain all constant maps.

So Chen spaces consist of a *set* of points equipped with the information which maps of sets from test domains into this set are regarded as smooth maps.

Chen spaces together with those morphisms of spaces $X \to Y$ between them which come from maps of the underlying sets $X_s \to Y_s$ form a closed subcategory

$$\mathrm{ChenSpaces} \subset \mathrm{Spaces}$$

of the category of all spaces. More details are in [6, 13].

3.1.2 L_∞-algebras and their classifying spaces

Definition 4 *A finite dimensional L_∞-algebra $\mathfrak{g}$ is a codifferential structure on a cofree coalgebra over a finite-dimensional $\mathbb{N}_+$-graded vector space V. By dualizing this corresponds bijectively to DGCAs whose underlying graded commutative algebra is freely generated over a finite dimensional $\mathbb{N}_+$-graded vector space V^*. These are called the corresponding Chevalley-Eilenberg algebras $\mathrm{CE}(\mathfrak{g})$.*

The mapping cone of the identity of $\mathrm{CE}(\mathfrak{g})$ is the Weil algebra $\mathrm{W}(\mathfrak{g})$. By the above it corresponds to an L_∞-algebra itself:

$$\mathrm{W}(\mathfrak{g}) =: \mathrm{CE}(\mathrm{inn}(\mathfrak{g}))\,.$$

Observation 1 *Since $\mathrm{CE}(\mathfrak{g})$ is trivial in degree 0, these Chevalley-Eilenberg algebras are naturally C^∞ DGCAs: the degree 0 part is the algebra of smooth functions on the point.*

Definition 5 (L_∞-algebra valued forms) *For $\mathfrak{g}$ an L_∞-algebra, $\mathfrak{g}$-valued forms on a space Y are morphisms*

$$(A, F_A) : \mathrm{W}(\mathfrak{g}) \to \Omega^\bullet(X)\,.$$

Flat $\mathfrak{g}$*-valued forms are morphisms*

$$A : \mathrm{CE}(\mathfrak{g}) \to \Omega^\bullet(X)\,.$$

We write

$$\Omega^\bullet(Y, \mathfrak{g}) := \mathrm{Hom}(\mathrm{W}(\mathfrak{g}), \Omega^\bullet(Y))$$

and

$$\Omega^\bullet_{\mathrm{flat}}(Y, \mathfrak{g}) := \mathrm{Hom}(\mathrm{CE}(\mathfrak{g}), \Omega^\bullet(Y))\,.$$

$$
\begin{array}{ccc}
\mathrm{CE}(\mathfrak{g}) & \twoheadleftarrow & \mathrm{W}(\mathfrak{g}) \\
\downarrow{\scriptstyle (A,0)} & & \downarrow{\scriptstyle (A,F_A)} \\
\Omega^\bullet(Y) & \overset{=}{\longrightarrow} & \Omega^\bullet(Y)
\end{array}
$$

Observation 2 (classifying spaces for $\mathfrak{g}$-valued differential forms) *By proposition 1 we have that $S(\mathrm{CE}(\mathfrak{g}))$ is the classifying space for $\mathfrak{g}$-valued differential forms:*

$$\Omega^\bullet(Y, \mathfrak{g}) \simeq \mathrm{Hom}(Y, S(\mathrm{CE}(\mathfrak{g}))) \,.$$

3.2 Various relations

3.2.1 Passing between spaces and DGCAs

Proposition 1 *The functors*

$$C^\infty : \ \mathrm{Spaces} \longleftrightarrow C^\infty\mathrm{Algebras} \ : S$$

and

$$\Omega^\bullet : \ \mathrm{Spaces} \longleftrightarrow \mathrm{DGCAs} \ : S$$

each form an adjunction.

So for all spaces X and C^∞-algebras A we have

$$\mathrm{Hom}(X, S(A)) \simeq \mathrm{Hom}(A, C^\infty(X)) \,,$$

and for all DGCA*s* A we have

$$\mathrm{Hom}(X, S(A)) \simeq \mathrm{Hom}(A, \Omega^\bullet(X)) \,.$$

Definition 6 (conjugation monad) *The monad*

$$S \circ C^\infty : \ \mathrm{Spaces} \longrightarrow \mathrm{Spaces}$$

we call the conjugation monad.

The unit

$$u : \mathrm{Id}_{\mathrm{Spaces}} \to S \circ C^\infty$$

of this monad is, by definition, an isomorphism on (Frölicher) *smooth* spaces X:

$$u_X : X \mapsto \mathrm{Hom}_{C^\infty \mathrm{Algebras}}(C^\infty(X), C^\infty(-)) \,.$$

Proposition 2 *There is a canonical map*

$$\mathrm{Hom}_{\mathrm{Spaces}}(X, Y) \longrightarrow \mathrm{Hom}_{C^\infty \mathrm{Algebras}}(C^\infty(X), C^\infty(Y)) \ .$$

If Y is a Frölicher smooth space then this map is an ismorphism.

Proof. The map is

$$\mathrm{Hom}(X, Y) \xrightarrow{\mathrm{Hom}(X, u_Y)} \mathrm{Hom}(X, S(C^\infty(Y))) \xrightarrow{\simeq} \mathrm{Hom}(C^\infty(Y), C^\infty(X)) \ .$$

□

3.2.2 The tensor product of C^∞DGCAs

For $C^\infty(X)$ and $C^\infty(Y)$ smooth function algebras on manifolds X and Y, their ordinary tensor product as vector spaces

$$C^\infty(X) \otimes C^\infty(Y) \subset C^\infty(X \times Y)$$

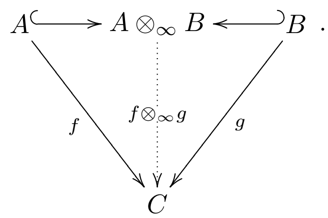
in general does not exhaust the space of smooth functions on $X \times Y$. Often such problems are dealt with by *completing* a tensor product.

If however we regard $C^\infty(X)$ not just as an object in CommAlgebras but as an object in C^∞Algebras, then this completed tensor product arises naturally simply as the canonical coproduct.

Definition 7 *Denote by*

$$\otimes_\infty : C^\infty\text{Algebras} \times C^\infty\text{Algebras} \to C^\infty\text{Algebras}$$

the coproduct in C^∞Algebras



Analogously for

$$\otimes_\infty : C^\infty\text{DGCAs} \times C^\infty\text{DGCAs} \to C^\infty\text{DGCAs}\,.$$

Proposition 3 *For all spaces X and Y we have*

$$C^\infty(X) \otimes_\infty C^\infty(Y) \simeq C^\infty(X \times Y)$$

Proof. First consider this for all $X = U, Y = V \in S$. Then for all $F \in C^\infty$Algebras we have

$\mathrm{Hom}(C^\infty(U) \otimes_\infty C^\infty(V), F)$	$\simeq$	$\mathrm{Hom}(C^\infty(U), F) \times \mathrm{Hom}(C^\infty(V), F)$	universal property of the coproduct
	$\simeq$	$F(U) \times F(U)$	Yoneda
	$\simeq$	$F(U \times V)$	since F is monoidal
	$\simeq$	$\mathrm{Hom}(C^\infty(U \times V), F)$	Yoneda

Since this is true for all F, again by the Yoneda lemma it follows that $C^\infty(U) \otimes_\infty C^\infty(V) \simeq C^\infty(U \times V)$.

From this the proposition follows by general facts about Day convolution. (** apparently, somehow...**)
□

Proposition 4 *The C^∞-algebra of smooth functions on $\mathbb{R}^n$ is* free *on n generators.*

This means that for any C^∞-algebra A we have

$$\mathrm{Hom}_{C^\infty\text{Algebras}}(C^\infty(\mathbb{R}^n), A) \simeq A^n\,.$$

3.3 Fundamental ∞-groupoids of spaces

For every space X we can form various flavours of path groupoids.

- The simplicial set of singular simplices in X

$$S^\bullet(X) = \{S^n(X) = \mathrm{Hom}_{\mathrm{Spaces}}(\Delta^n, X)\}$$

  plays the role of the weak fundamental ∞-groupoid $\Pi_\infty^{\mathrm{wk}}(X)$ of X.

- For each integer n we can form a strict globular n-groupoid $\Pi_n^{\mathrm{str}}(X)$, the strict fundamental n-groupoid of X.

But it is useful to observe that even without forming n-groupoids this way, a space itself, in our sense, behaves a lot like an ∞-category already:

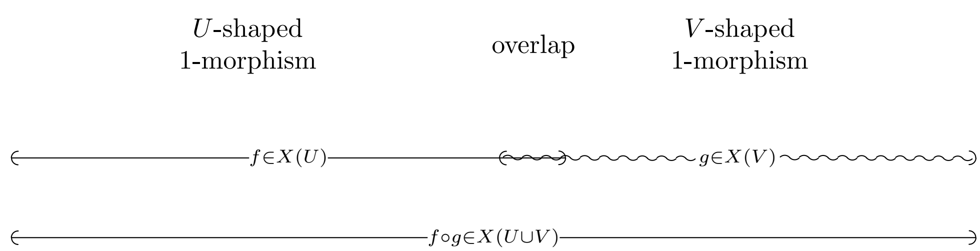


Figure 1: **Spaces and ∞-groupoids.** A sheaf X on open subsets of $\mathbb{R}^n$ behaves not entirely unlike a presheaf on Δ (a simplicial set) satisfying the Kan condition: for each object $U \subset \mathbb{R}^k$ there is a collection $X(U)$ of "U-shaped k-morphisms" and the sheaf condition says that whenever these overlap with V-shaped k-morphisms, there is a (unique) composite $(U \cup V)$-shaped k-morphism. We see that this is more than a faint analogy when discussing integration of L_∞-algebras in 4.1.

for X a space and $U \subset \mathbb{R}^k$ an open subset, an element in $X(U)$ is like a "U-shaped k-morphism". Given another V-shaped k-morphism we can ask whether both overlap, i.e. whether there is "source-target matching" between both. This is the case if their restriction to $U \cap V$ coincides. If it does then, by the fact that a space is a sheaf, there is guaranteed to be a unique $(U \cup V)$-shaped element in X. This we can regard as the *composite* k-morphism obtained by composing the U-shaped and the V-shaped k-morphism we started with. See figure 1.

Therefore one can take the standpoint that a space X *is* already nothing but its own fundamental ∞-groupoid: the relation between spaces and ∞-groupoids is blurred to a tautology from this point of view.

We shall come back to this later in 4.1.

4 Integration

4.1 Integration of L_∞-algebras

Definition 8 *Fix some notion of Lie ∞-groupoids and the corresponding notion of the fundamental Lie ∞-groupoid $\Pi_\infty(X)$ of any space X.*

Then the Lie ∞-groupoid integrating an L_∞-algebra $\mathfrak{g}$ is

$$\mathbf{B}(\int \mathfrak{g}) := \Pi_\infty S(\mathrm{CE}(\mathfrak{g}))\,.$$

Examples. Let $\mathfrak{g}$ be an ordinary Lie algebra and $\Pi_1(X)$ the strict fundamental 1-groupoid of a space X (morphisms are homotopy classes of paths). Let G be the simply connected Lie group integrating $\mathfrak{g}$. Then

$$\Pi_1(S(\mathrm{CE}(\mathfrak{g}))) = \mathbf{B}G\,,$$

where the right hand side denotes the strict one object 1-groupoid obtained from G.

Now let $\mathfrak{g}$ be an ordinary Lie algebra with a bilinear invariant form on it and let μ be the associated canonical Lie algebra 3-cocycle. The corresponding String Lie 2-algebra is $\mathfrak{g}_\mu$. Let $\Pi_2(X)$ be the strict fundamental 2-groupoid of a space X: morphisms are *thin* homotopy classes of paths and 2-morphisms are homotopy classes of paths [10].

Then, I am claiming, the 2-group G_μ defined by

$$\mathbf{B}G_2 := \Pi_2(S(\mathrm{CE}(\mathfrak{g}_\mu)))$$

is essentially the strict version of the String Lie 2-group presented in [1], only that the horizontal composition of paths is not pointwise multiplication, but concatenation. This is, I am claiming, the strict 2-group secretly underlying the discussion in [?].

Forming instead $\Pi_\infty^{\mathrm{wk}}(S(\mathrm{CE}(\mathfrak{g})))$ leads to the integration discussed in [5].

4.2 Integration and basic forms on mapping spaces

Definition 9 (integral of a $\mathfrak{g}$-valued form) *Let $\mathfrak{g}$ be any L_∞-algebra and fix a $\mathfrak{g}$-valued differential form*

$$\Omega^\bullet(Y) \xleftarrow{(A,F_A)} \mathrm{W}(\mathfrak{g})\ .$$

For any smooth space Σ, we say that the integral of A over Σ *is the morphism*

$$\int_\Sigma A\ :\quad \Omega^\bullet_{\mathrm{basic}}(\hom(\Sigma, S(\mathrm{W}(\mathfrak{g})))) \longrightarrow \Omega^\bullet(\hom(\Sigma, X))$$

in

$$\begin{array}{ccc} & & \Omega^\bullet(\hom(\Sigma, S(\mathrm{CE}(\mathfrak{g})))) \\ & & \uparrow \\ \Omega^\bullet(\hom(\Sigma, X)) & \xleftarrow{\Omega^\bullet(\hom(\Sigma, S(A,F_A)))} & \Omega^\bullet(\hom(\Sigma, S(\mathrm{W}(\mathfrak{g})))) \\ & \nwarrow^{\int_\Sigma A} & \uparrow \\ & & \Omega^\bullet_{\mathrm{basic}}(\hom(\Sigma, S(\mathrm{W}(\mathfrak{g})))) \end{array}$$

Example. Elsewhere I sketched the proof of the obvious consistency condition: let $\mathfrak{g} = b^{n-1}\mathfrak{u}(1)$. Then A is an ordinary n-form on X. Let Σ be n-dimensional. Then $\int_\Sigma A$ coincides with the ordinary integral of A over Σ.

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