transport of sections

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Abstract

Given a 2-vector 2-transport, we identitfy those notions that are necessary to define the holonomy over a disk with specified boundary condition and with up to two given boundary insertions.

Definition 1 *The* **2-point** *is the category*

$$p_2 \equiv \{ \bullet \longrightarrow \circ \}$$

consisting of two objects and a single nontrivial morphism.

Definition 2 *The category of 2-point* **cobordisms** *is the 2-functor 2-category*

$$\mathrm{Cob}_2 \equiv [p_2, \mathcal{P}_2(X)] .$$

The discrete category over the collection of objects is the **configuration space**

$$\mathrm{Conf}_2 \equiv \mathrm{Disc}(\mathrm{Obj}(\mathrm{Conf}_2))$$

of the 2-point.

Let $\mathcal{C}$ be a monoidal category, and let

$$T \equiv \mathrm{Bim}(\mathcal{C})$$

be the 2-category of bimodules internal to $\mathcal{C}$.

Fix a 2-transport

$$\mathrm{tra} : \mathcal{P}_2(X) \to T .$$

Denote by

$$\mathcal{P}_2(X) \to \mathrm{Id}_{\mathbb{1}}$$

the unique 2-transport on $\mathcal{P}_2(X)$ that sends everything to the identity on the tensor unit $\mathbb{1}$.

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Definition 3 *A* **section** *of the 2-transport* tra *is a morphism*

$$\mathrm{Conf}_2 \; \underset{\mathrm{tra}_*}{\overset{(\mathcal{P}_2(X)\to\mathrm{Id}_{1\!\!1})_*}{\Downarrow e}} \; [p_2, T] \; .$$

Accordingly, the **space of sections** *is the category*

$$\Gamma \equiv \left[(\mathcal{P}_2(X) \to \mathrm{Id}_{1\!\!1})_*, \mathrm{tra}_*\right].$$

A **cosection** *of the 2-transport* tra *is a morphism*

$$\mathrm{Conf}_2 \; \underset{(\mathcal{P}_2(X)\to\mathrm{Id}_{1\!\!1})_*}{\overset{\mathrm{tra}_*}{\Downarrow f}} \; [p_2, T] \; .$$

Accordingly, the **space of cosections** *is the category*

$$\Gamma \equiv \left[\mathrm{tra}_*, (\mathcal{P}_2(X) \to \mathrm{Id}_{1\!\!1})_*\right].$$

Notice that, since Conf_2 is a discrete category, a section e is a collection of morphisms

$$p_2 \; \underset{\gamma^*\mathrm{tra}}{\overset{\gamma^*(\mathcal{P}_2(X)\to\mathrm{Id}_{1\!\!1})}{\Downarrow e(\gamma)}} \; T \; ,$$

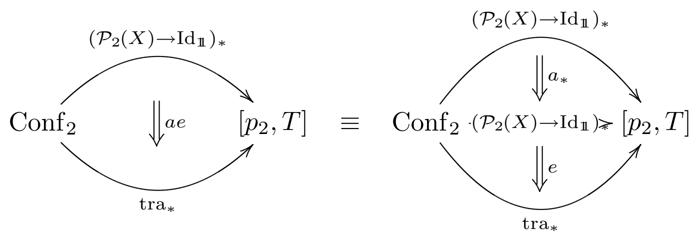
one for each $\gamma : p_2 \to \mathcal{P}_2(X)$.

Definition 4 *The* **algebra of observables** *is the monoid*

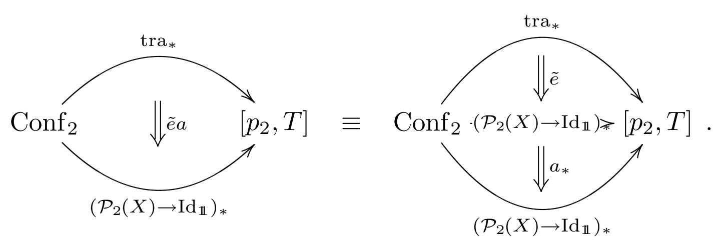
$$A \equiv \mathrm{End}\left(\mathcal{P}_2(X) \to \mathrm{Id}_{1\!\!1}\right) \, .$$

Notice that this is the monoid of bundles with connection on X whose fibers are objects of C. It is commutative in as far as C is braided.

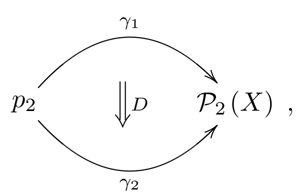
Definition 5 *The space of sections* Γ *is equipped with a left* A*-action, and the space of cosections with a right* A*-action in the obvious way, by pre- and* *postcomposition, respectively:*



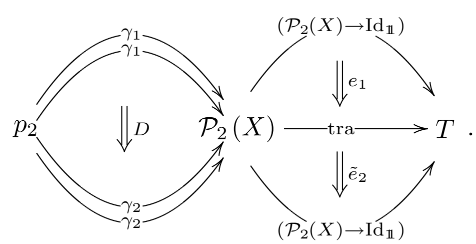
and



Definition 6 *A* **two-point disk transport** *associated to a cobordism*

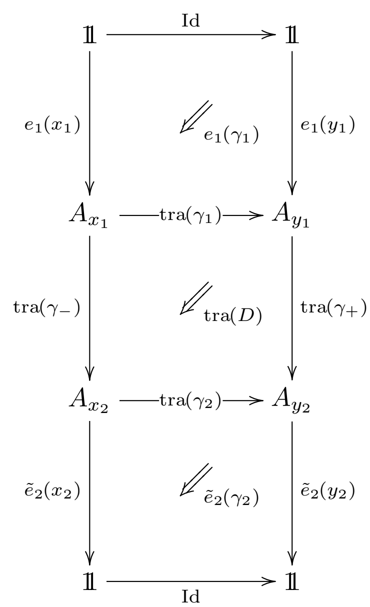


as well as to a section e_1 *and a cosection* $\tilde{e}_2$ *is the morphism*



Example 1

This two-point disk transport is given by a 2-morphism in T of the form



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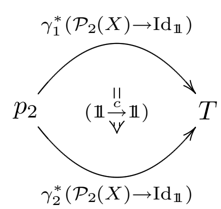
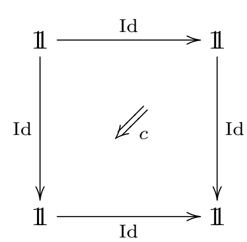
This describes a section coming in, propagating along D, and being projected on the section coming out.

Definition 7 *Let*



be the peusonatural transformation given by the 2-morphism



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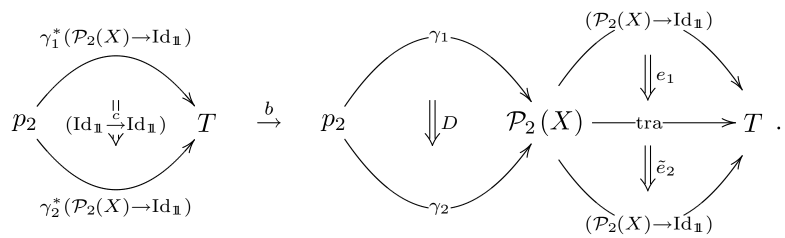
A **boundary condition** *b for a two-point disk transport is a morphism*



Here $\mathrm{Id}_{\mathbb{1}} \xrightarrow{c} \mathrm{Id}_{\mathbb{1}}$ *is the* **two-point disk holonomy** *of the two-point disk transport for the given boundary condition b.*

Example 2

Assume that everything takes values not in arbitrary bimodules, but just in right induced bimodules

$$\mathrm{RIBim}(\mathcal{C}) \subset \mathrm{Bim}(\mathcal{C}) ,$$

and that the sections involved are such that

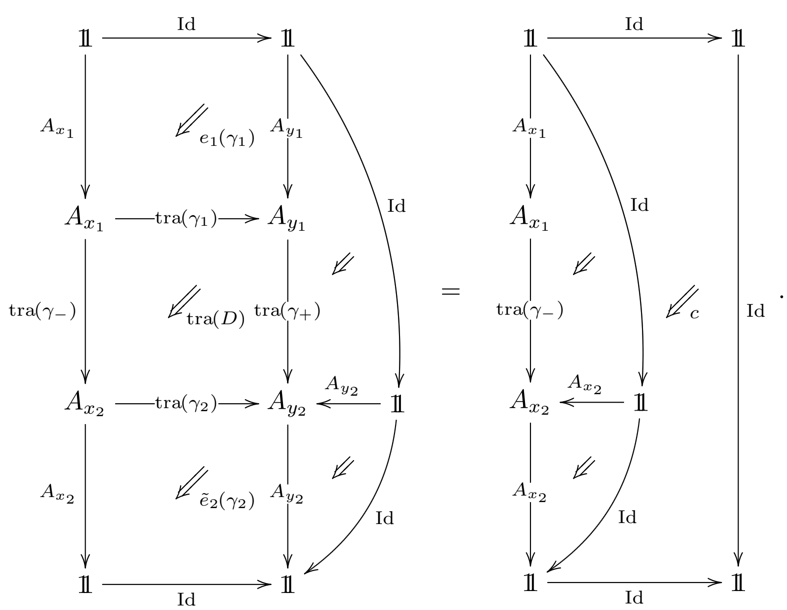
$$(\mathbb{1} \xrightarrow{e_1(x)} A_x) = (\mathbb{1} \xrightarrow{A_x} A_x) ,$$

as well as

$$(A_x \xrightarrow{\tilde{e}_2(x)} \mathbb{1}) = (A_x \xrightarrow{A_x} \mathbb{1}) ,$$

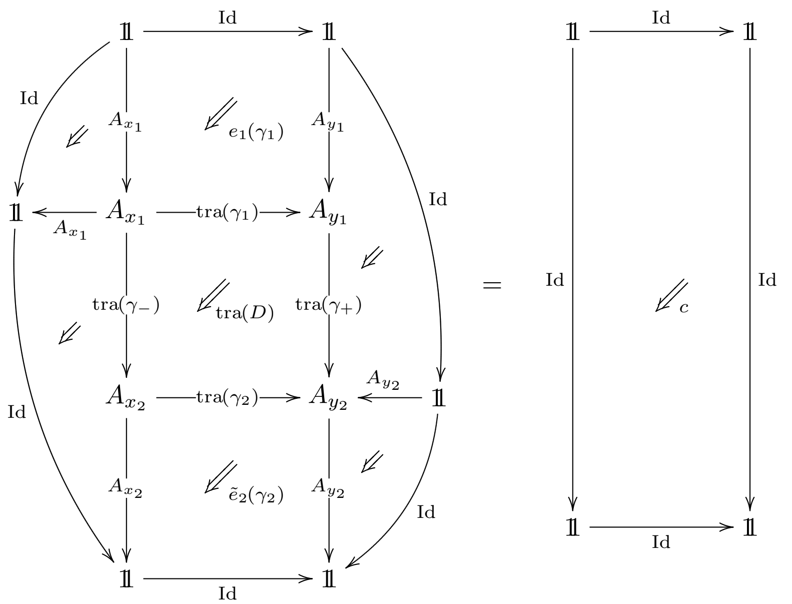
for all $x \in X$.

Then a boundary condition b for this is given by the modification of pseudo-natural transformations that looks like



The unlabeled 2-morphisms here are the canonical ones.

This modification has a one-sided inverse, which allows to deduce that the two-point disk holonomy is



If we now let tra be the 2-connection on a $U(1)$-gerbe and remove the two insertion points by setting $\gamma_1 = \mathrm{Id}$ and $\gamma_2 = \mathrm{Id}$, then this reproduces the diagram for the disk holonomy of a $U(1)$-gerbe.