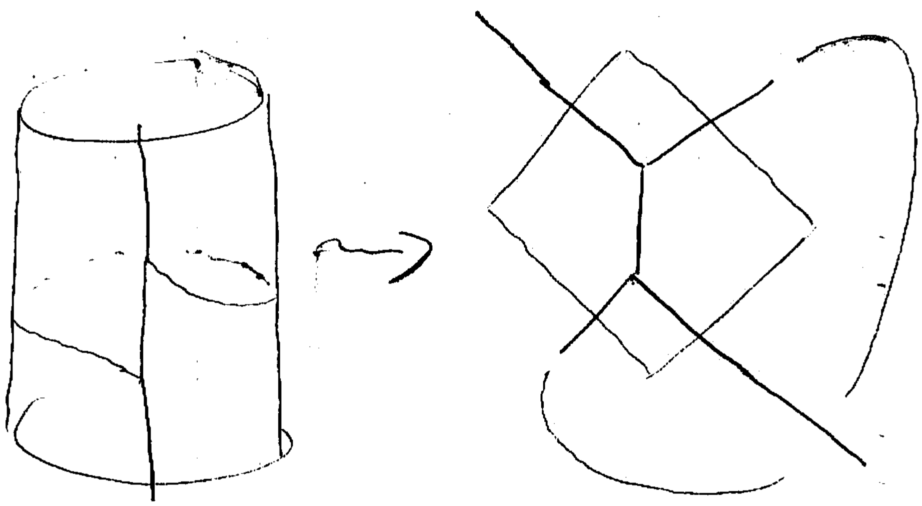
Third part

- on which we glue what we had sliced

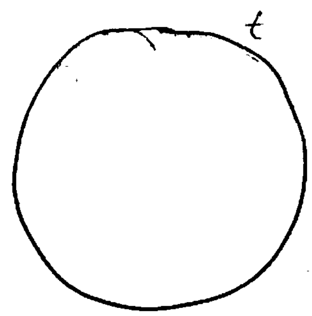


- and finally see the need for

Frobenius algebras internal to ribbon categories

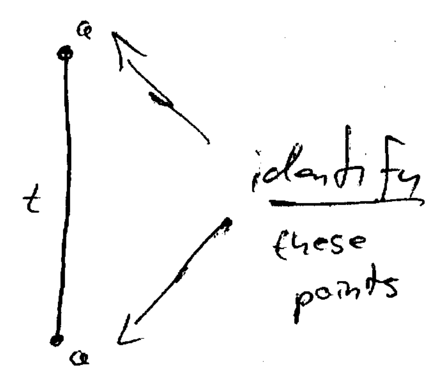
example for <u>what we are trying to do</u> in the case of $n=1$ QFT
(quantum mechanics)

start with the circle



$= S^1$

cut it open $\rightsquigarrow$

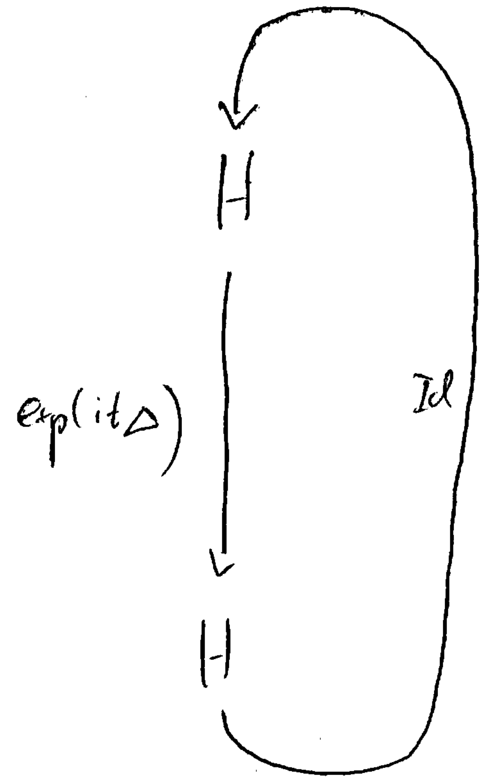


apply functor $U : 1\mathrm{Cob}_{Riem} \longrightarrow \mathrm{Hilb}$

$$U : \left(t \, \Big| \right) \longmapsto \begin{array}{c} H \\ \big\downarrow {\scriptstyle \exp(itA)} \\ H \end{array}$$

then implement the identification by taking <u>a trace</u>.

Trace:

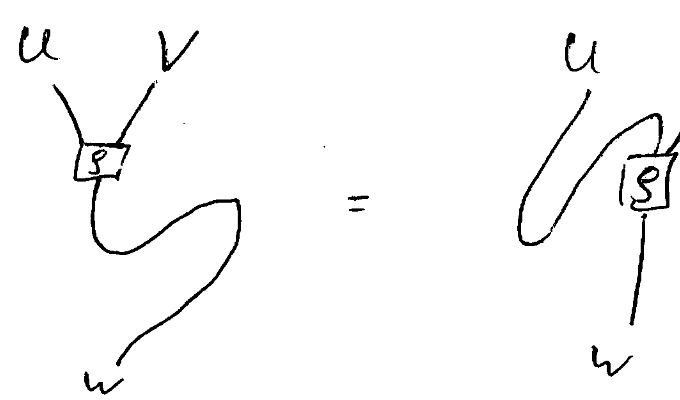


$$=\quad \begin{tikzcd} \mathbb{C} \arrow[d] & 1 \arrow[d, mapsto] \\ H \otimes H^* \arrow[d, "\exp(it\Delta)\ \otimes\ Id"'] & Id \\ H \otimes H^* \arrow[d, "\text{contraction}"] & \\ \mathbb{C} & \end{tikzcd}$$

what we are doing here makes use of the fact that Hilb (Vect) is a <u>monoidal category with duals</u>

this means that morphisms behave like <u>strings on a plane</u>

for instance



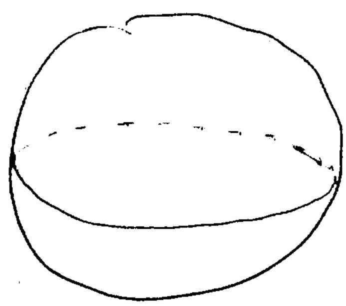
etc.

so in particular, the trace ensures that our choice of cutting the circle becomes immaterial

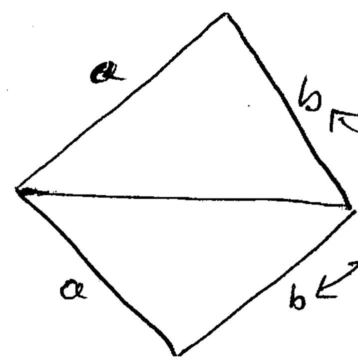
let's try the same for surfaces:

Examples:

1) **sphere** S^2 $\rightsquigarrow$ triangulated sphere

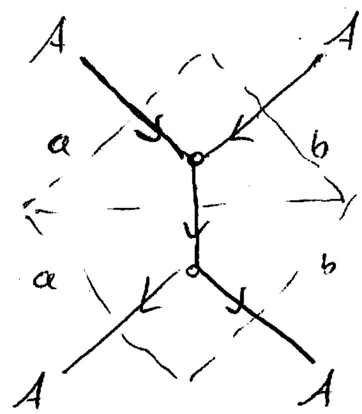


$\rightsquigarrow$



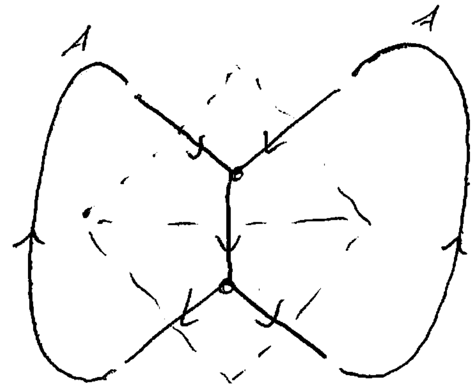
identify these edges

$\rightsquigarrow$



decorated triangulation

$\rightsquigarrow$

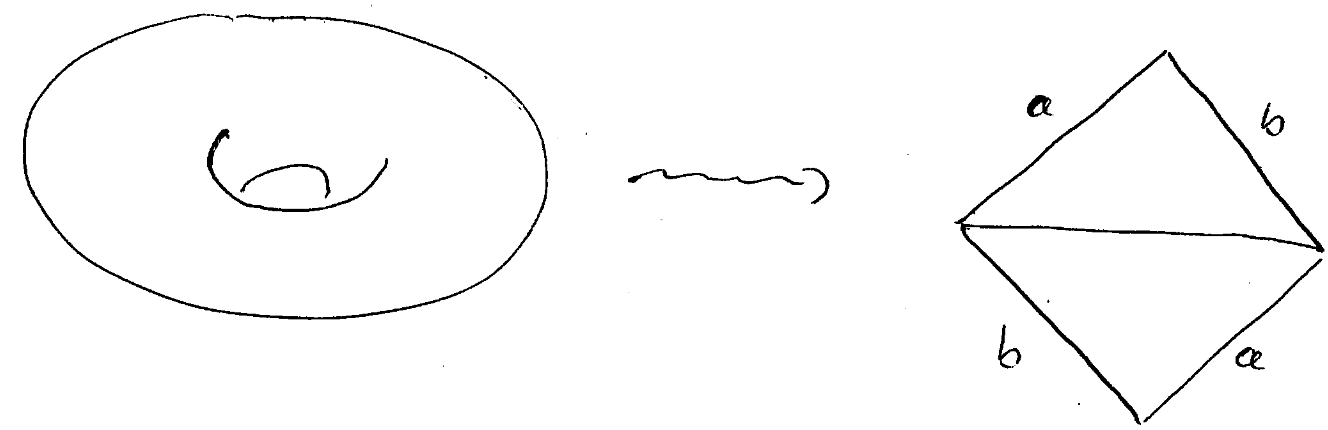


trace over decorated triangulation

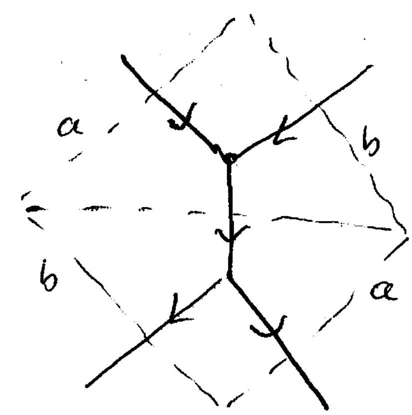
okay, nothing new, but now ...

2) Torus T^2

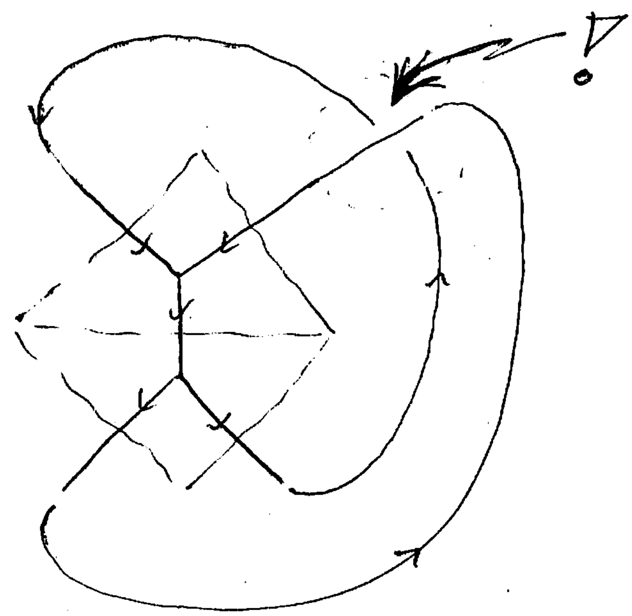
triangulated torus



decorated triangulation



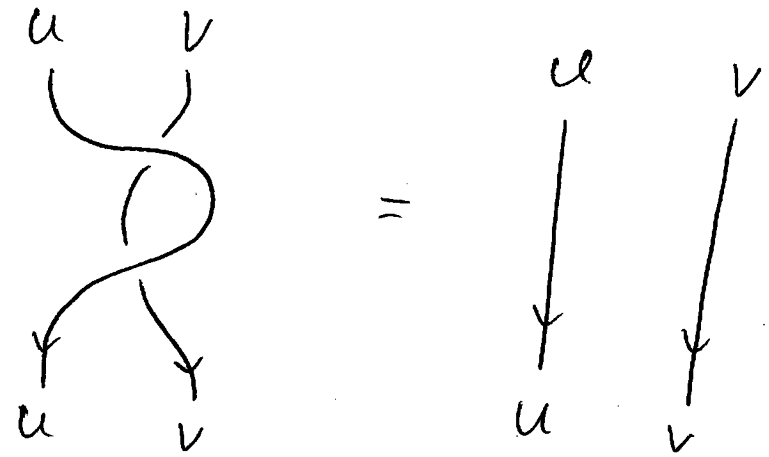
trace over decorated triangulation



so here we need more structure...

to glue our surfaces
by taking traces, we need
a <u>braided</u> monoidal category
with duals

for instance:

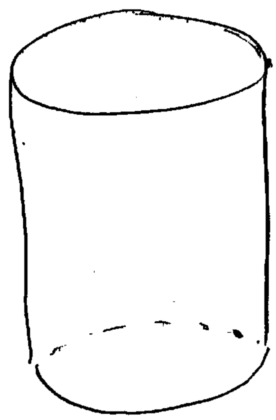


whose morphisms behave like
<u>strings in 3-dimensional space</u>

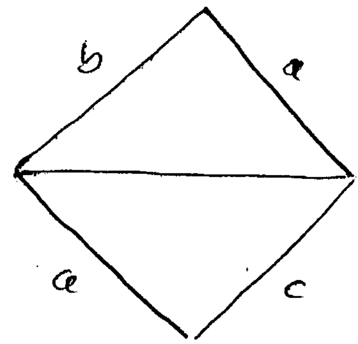
more generally, we can allow
these morphisms to behave like
<u>ribbons</u> in 3-dimensional space

3)

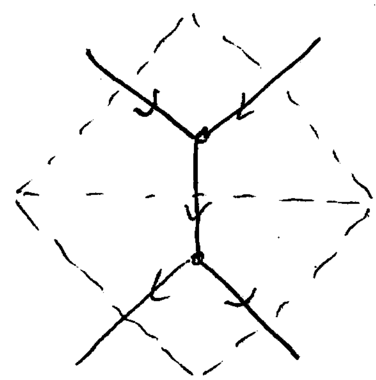
cylinder / annulus $\rightsquigarrow$ triangulated cylinder



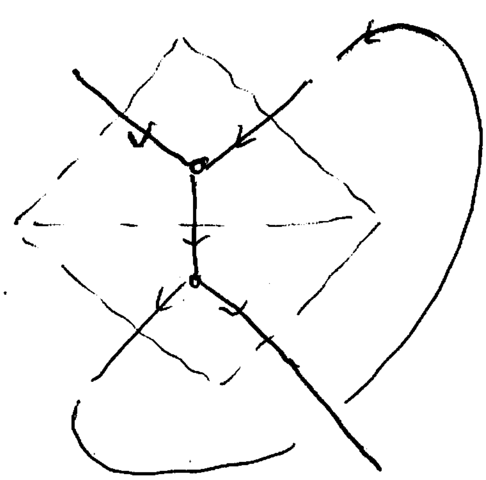
$\rightsquigarrow$



decorated triangulation $\rightsquigarrow$

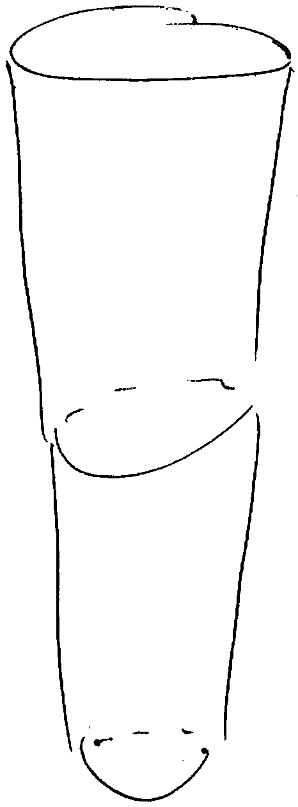


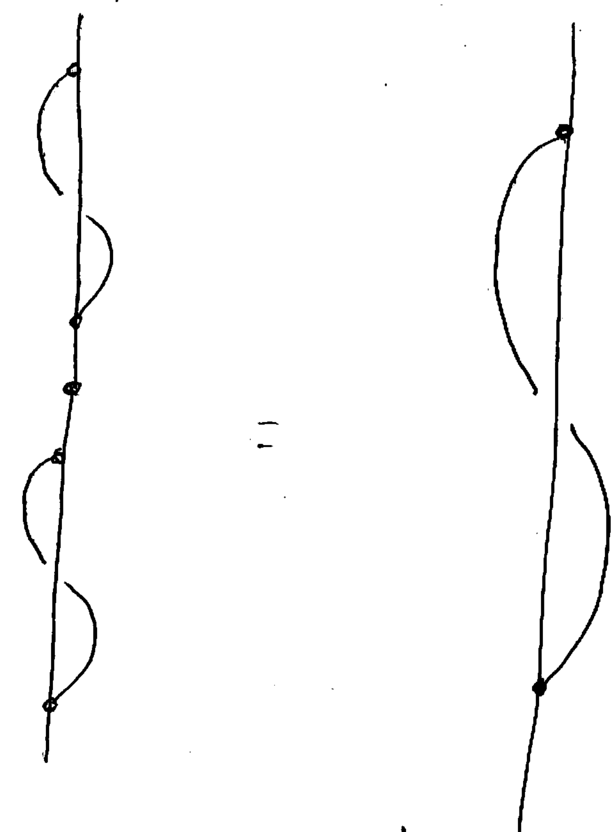
trace over decorated triangulation $\rightsquigarrow$



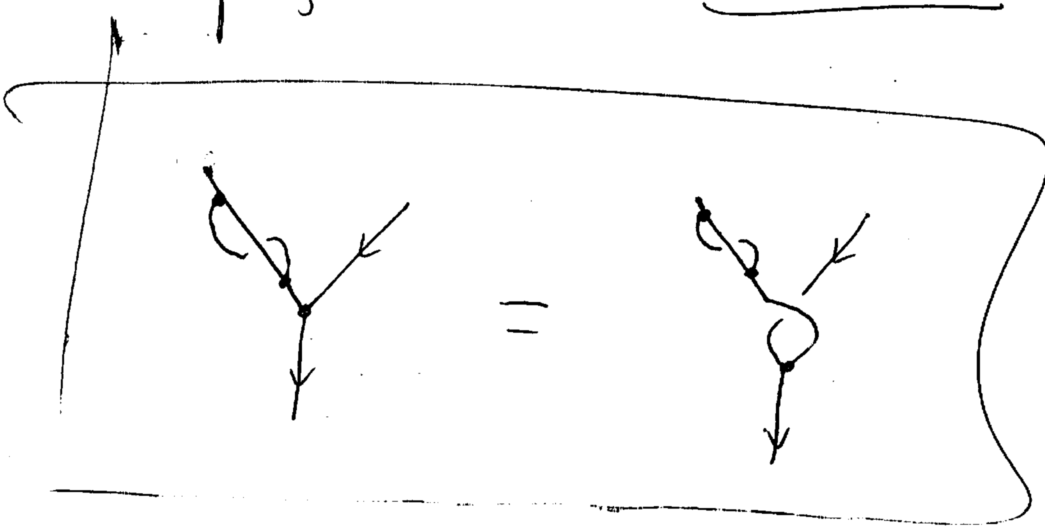
notice the following properties of the cylinder decoration

it's a projector





it projects into the center of the Frob. algebra



so we see how the global picture is recovered ...