End $(\Sigma(C_2))$-2-Transport

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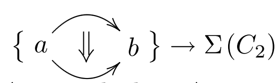
Abstract

Principal 2-transport with respect to a 2-group G_2 involves the automorphisms 3-group $\mathrm{AUT}(G_2)$. Propagation in 2-dimensional field theory is similar, but uses a 2-monoid C_2 instead of a 2-group G_2. Hence the 3-monoid $\mathrm{END}(C_2)$ should play a role here.

We compute the inner part of $\mathrm{END}(C_2)$ for C_2 a braided monoidal category with duals. Then we show that bimodule homomorphisms internal to $\mathrm{END}(C_2)$ give rise to morphisms of left- and right-induced bimodules in C_2.

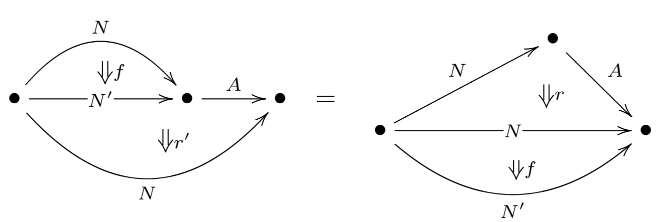
In our applications, C_2 will be abelian. Therefore we call a monad in $\Sigma(C_2)$ an algebra internal to C_2.

A lax functor



is the same as an algebra A_a, an algebra A_b, two A_a-A_b bimodules and a bimodule homomorphism between these, all internal to C_2.

Writing r for the right action on a bimodule, respect for the right module structure is in particular expressed by



We want to understand what happens with this situation as we pass from $\Sigma(C_2)$ to $\mathrm{End}(\Sigma(\mathrm{C}_2))$.

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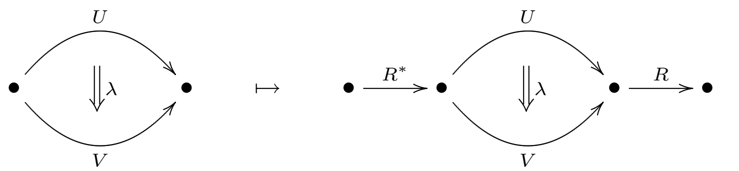
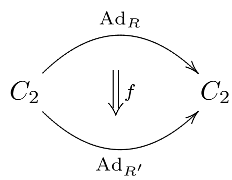
The inner part of $\mathrm{End}(\Sigma(C_2))$**.** For $R \in \mathrm{Obj}(C_2)$, consider the inner endomorphism of $\Sigma(C_2)$ obtained by conjugation with R

$$\mathrm{Ad}_R \quad : \quad C_2 \quad \to \quad C_2$$

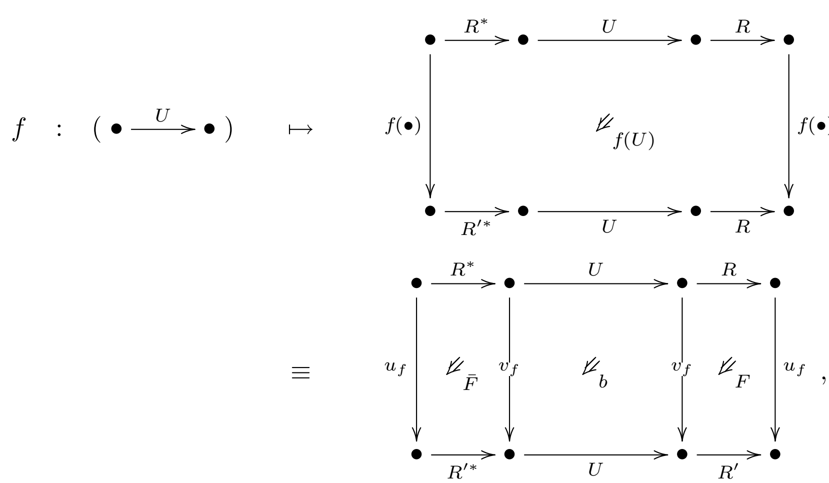


Using duality on objects in C_2, this functor has lax and op-lax respect for horizontal composition.

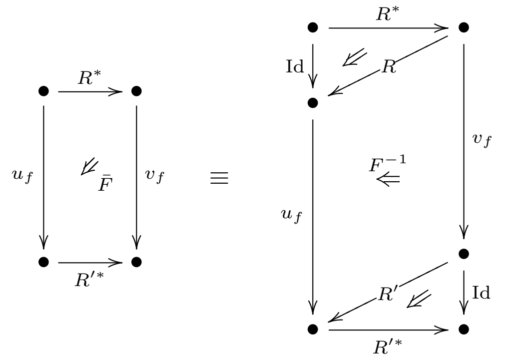
Between two such morphism, we have 2-morphisms



given by pseudonatural transformations which are represented by functorial assignments

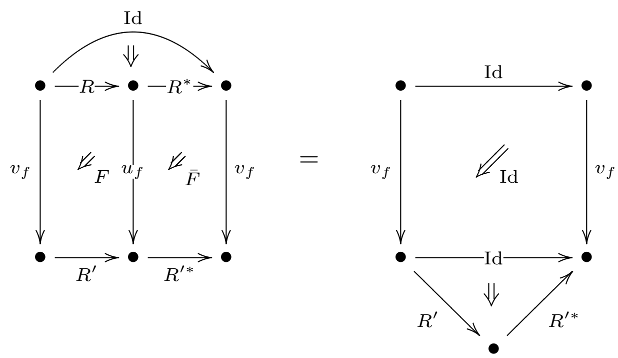


where

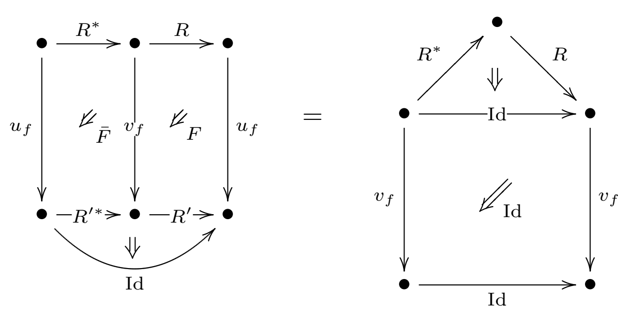


Here and in the following the 2-morphism b denotes the respective braiding operation.

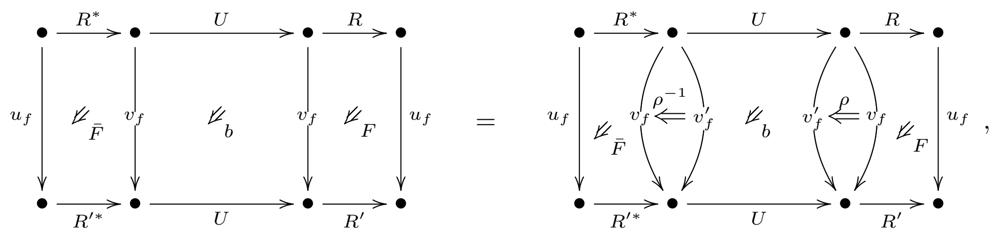
This f respects horizontal composition using the op-lax compositor



and it respects the identity in an op-lax way

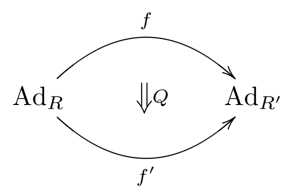


In the above form, f depends only on the isomorphism class of v_f, since

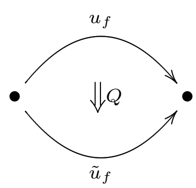


by the functoriality of the braiding.

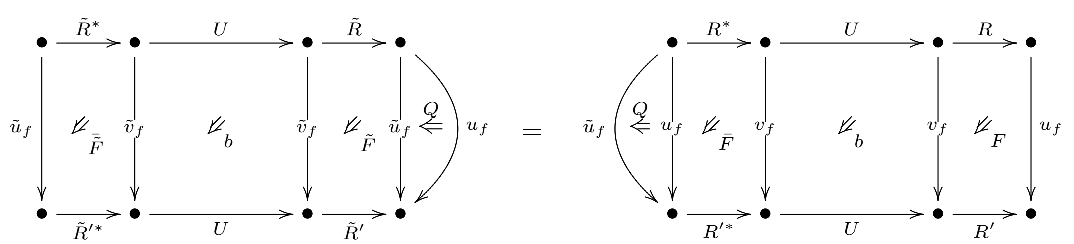
On the other hand, a 3-morphism



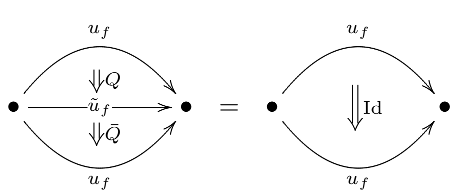
being a modification of pseudonatural transformations, is a 2-cell



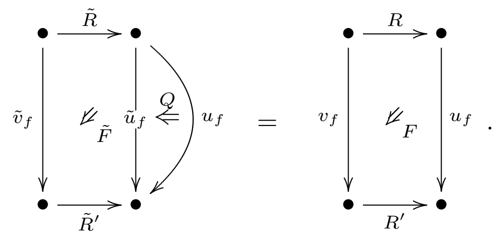
such that



If Q has at least a one-sided inverse

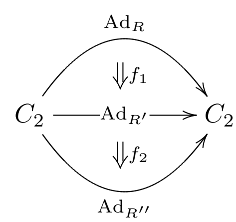


this condition is equivalent to

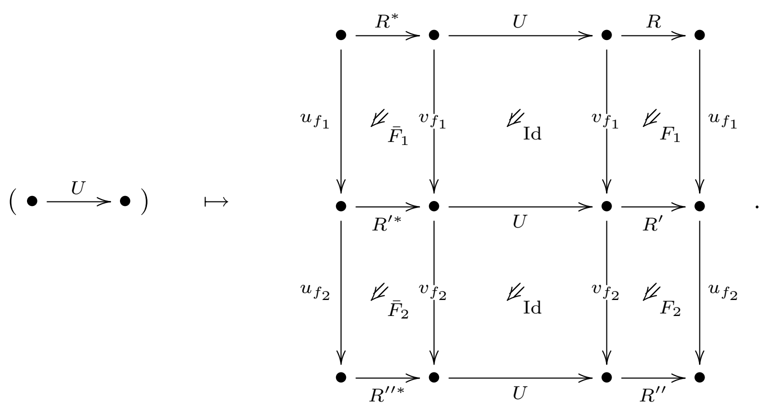


This is analogous to the action of isomorphisms on v_f above.

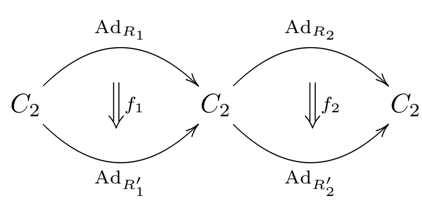
The vertical composition of two such 2-morphisms



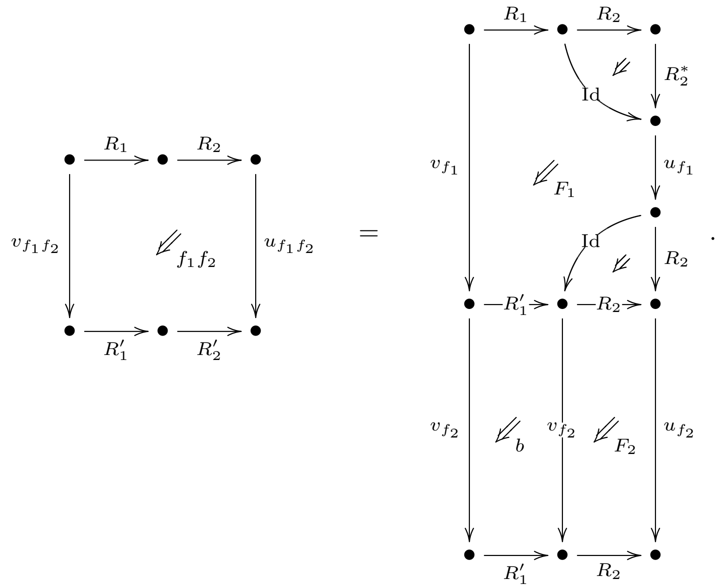
is represented by the assignment



We find the horizontal composition



by whiskering with identity 2-cells and applying vertical composition. The result should be

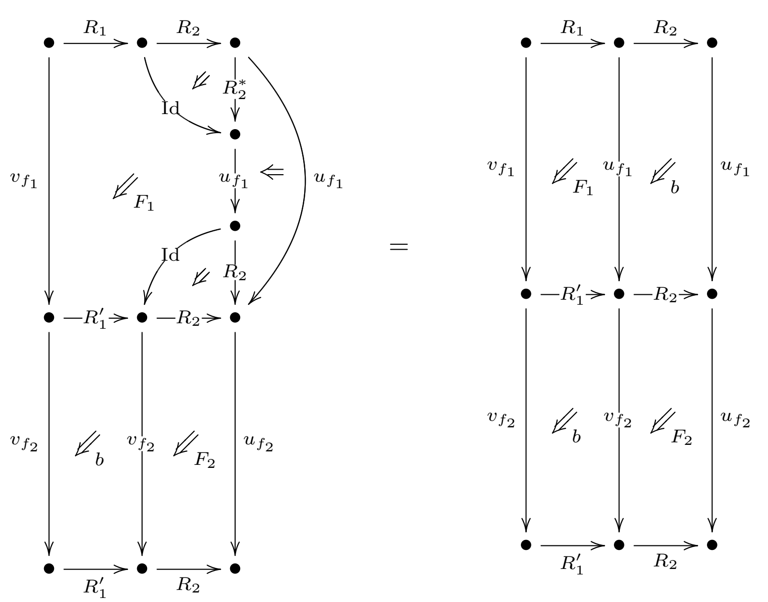


For our applications, we can assume all the units

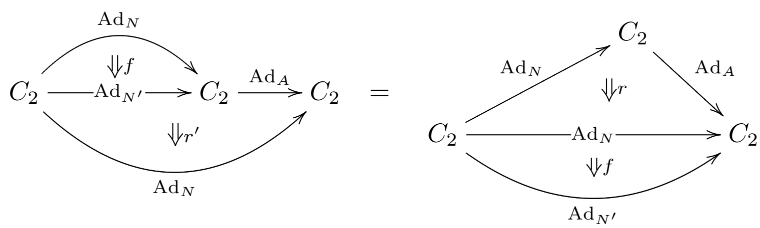
$$\mathrm{Id} \to R^* \otimes R$$

of the dualities in C_2 to have right inverses.

Then there is a modification with one-sided inverse given by $u_f \to R^* \otimes R \otimes u_f \overset{R^* \otimes b}{\to} R^* \otimes u_f \otimes R$ with



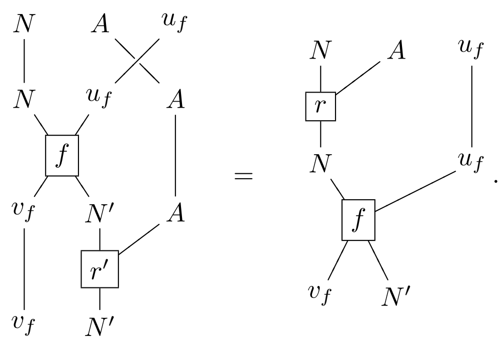
Bimodule homomorphisms in $\mathrm{End}(\Sigma(C_2))$**.** Using the above, we can work out the diagram



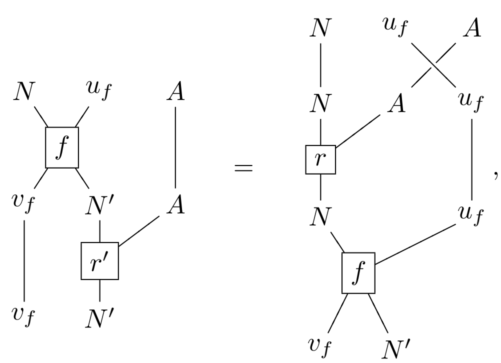
in $\Sigma(\mathrm{End}(C_2))$.

For our applications we are interested in the case where the right actions r are pseudonatural transformations with $u_r = \mathrm{Id}$ and $v_r = \mathrm{Id}$.

In this case, using our results on the nature of inner 2-morphisms in $\mathrm{End}(\Sigma(C_2))$, we find that the above equation says in terms of string diagrams in C_2 that



This is equivalent to



which says that a morphism of bimodules in $\mathrm{End}(\Sigma(C_2))$ is a morphism of left- and right-induced bimodules in $\Sigma(C_2)$.