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Mathias Schacht

On the Regularity Method for Hypergraphs

By

Mathias Schacht
Doctor of Philosophy

Department of Mathematics and Computer Science

Vojtěch Rödl
Advisor

Dwight Duffus
Committee Member

Tomasz Łuczak
Committee Member

Endre Szemerédi
Committee Member

Accepted:

Dean of the Graduate School

Date

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An Abstract of
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Abstract

We present alternative proofs of density versions of some combinatorial partition theorems originally obtained by E. Szemerédi, H. Furstenberg and Y. Katznelson. The proofs presented here are based on an extension of the *Regularity Method* from graphs to k -uniform hypergraphs.

Szemerédi's Regularity Lemma for graphs asserts that every graph can be decomposed into relatively few random-like subgraphs. This random-like behavior enables one to find and enumerate subgraphs of a given isomorphism type. This observation is called Counting Lemma. The interplay of Szemerédi's Regularity Lemma and the Counting Lemma, referred to as the Regularity Method for graphs, has many applications in the area of extremal graph theory.

Recently, V. Rödl and J. Skokan (based on earlier work of P. Frankl and V. Rödl) generalized Szemerédi's Regularity Lemma from graphs to k -uniform hypergraphs for arbitrary $k \geq 2$. In the main part of this dissertation we prove a Counting Lemma accompanying the Rödl–Skokan Hypergraph Regularity Lemma. Both lemmas together establish a generalization of the Regularity Method from graphs to k -uniform hypergraphs.

A similar extension of the Regularity Method was independently and alternatively obtained by W. T. Gowers. His results can also be used to derive alternative proofs of the density theorems mentioned above.

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dedicated to my family and my friends