Fachbereich Mathematik
Lecturer: Prof. Mathias Schacht
Assistant: Dr. Christian Reiher

Universität Hamburg
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February 9th, 2021

Graph Theory 2

extra set

due February 16th, 10am

https://bit.ly/3tGonps

Exercise 1 [1 point]

Can the tree-width of a subdivision of a graph G be smaller than the tree-width of G? Can it be larger?

Exercise 2 [1 point]

Determine the tree-width of cliques, cycles, trees, and grids. (For the lower bound for grids you may use the duality theorem of Seymour and Thomas.)

Exercise 3 [1 point]

Show that every triangle-free graph $G = (V, E)$ with minimum degree $\delta(G) > 2|V|/5$ is bipartite and show that $2/5$ is optimal for that assertion.

Exercise 4 [1 point]

Determining the chromatic number of a graph $G = (V, E)$ and finding an optimal colouring are NP-complete problems. In fact, already deciding if $\chi(G) = 3$ is NP-complete. Roughly speaking, this means that it is not expected that there is an algorithm that solves these problems with a running time polynomial in $|V|$.

(i) Describe a polynomial time algorithm that determines if a graph G is bipartite and which outputs a 2-colouring if $\chi(G) \leqslant 2$.

(ii) Describe an algorithm that given a 3-chromatic graph $G = (V, E)$, finds a $10\sqrt{|V|}$-colouring in polynomial time.

Prove the correctness and explain the running time of your algorithms.