

Graph Theory 2

9th problem set

due February 2nd, 10am

<https://bit.ly/3t48gla>

Exercise 1 [1 point]

Prove an upper bound for the off-diagonal graph Ramsey number $R^{(2)}(K_3, K_n)$ that is polynomial in n .

Exercise 2 [1 point]

Prove $2^r < R^{(2)}(3; r) < 3r!$ for the r -colour Ramsey number for graph triangles.

Exercise 3 [1 point]

Show that for any given graph H there exists a graph $G = (V, E)$ such that for every vertex 2-colouring of V there exists an induced copy of H in G with all vertices of the same colour.

Exercise 4 [1 point]

A family $\mathcal{F}$ of sets is called a *weak Δ -system* if every two sets have intersections of the same size and it is a Δ -system if every two sets have in fact the same intersection.

- (i) Show that for all integers $m, k \geq 2$ there exists an M such that every family $\mathcal{F}$ of k -element sets with $|\mathcal{F}| = M$ contains a weak Δ -system $\mathcal{F}' \subseteq \mathcal{F}$ with $|\mathcal{F}'| \geq m$.
- (ii) Show that for all integers $m, k \geq 2$ there exists an M such that every weak Δ -system $\mathcal{F}$ of k -element sets with $|\mathcal{F}| = M$ contains a Δ -system $\mathcal{F}' \subseteq \mathcal{F}$ with $|\mathcal{F}'| \geq m$.