Fachbereich Mathematik
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Universität Hamburg
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Graph Theory 2

6th problem set

due January 12th, 10am

https://bit.ly/3rVxAth

Exercise 1 [1 point]

Show directly, without using the *6-flow theorem* (Theorem 6.6.1), that the flow number $\varphi(G)$ is finite for every bridgeless multigraph G.

Can you show an universal upper bound on $\varphi(G)$ that is independent of G?

Exercise 2 [1 point]

Prove that a plane triangulation is 3-colourable if and only if all its vertices have even degree.

Exercise 3 [1 point]

In class we defined a pair (X, Y) in a graph $G = (V, E)$ for nonempty, disjoint sets X, $Y \subseteq V$ to be (ε, d)*-regular pair*, if for all $X' \subseteq X$ and $Y' \subseteq Y$ we have

$$\left|e_G(X', Y') - d\,|X'||Y'|\right| \leqslant \varepsilon|X||Y|\,.$$

Show that this definition is equivalent to the one in the textbook [Diestel, *Graph Theory*, 5th ed., page 187/188]. More precisely, show that for every $\varepsilon > 0$ there is some $\delta > 0$ such that every (δ, d)-regular pair satisfying the definition above is ε-regular following the definition from the textbook and in the opposite direction that every δ-regular pair satisfying the textbook definition is (ε, d)-regular as defined above with $d = d(X, Y) := \frac{e_G(X,Y)}{|X||Y|}$.

Exercise 4 [1 point]

The *triangle counting lemma* asserts that for every $\gamma > 0$ there exists $\varepsilon > 0$ such that if all three bipartite pairs (V_i, V_j) of a tripartite graph $G = (V_1 \cup V_2 \cup V_3, E)$ are (ε, d_{ij})-regular for some $d_{ij} \in [0, 1]$ for $1 \leqslant i < j \leqslant 3$, then the number of triangles in G is in the interval $(d_{12}d_{13}d_{23} \pm \gamma)|V_1||V_2||V_3|$.

Is a similar assertion true, if only two, or if only one of the three pairs is ε-regular? Give a proof or a counterexample for those assertions.