Fachbereich Mathematik | Universität Hamburg
Lecturer: Prof. Mathias Schacht | Fall 2020/21
Assistant: Dr. Christian Reiher | December 8th, 2020

Graph Theory 2

5th problem set

due December 15th, 10am

https://bit.ly/39UjhOQ

Exercise 1 [1 point]

Consider the following recursive definition of a function $\varphi_G \colon \mathbb{R} \longrightarrow \mathbb{R}$ associated to a graph $G = (V, E)$ with at least one vertex: For $E = \varnothing$ we set $\varphi_G(x) = x^{|V|}$ and if $e \in E$ then we set

$$\varphi_G = \varphi_{G-e} - \varphi_{G/e},$$

where $G - e$ is the graph obtained from G by removing the edge e and G/e is obtained from G by contracting the edge e.

(*i*) Find a graph theoretic interpretation for φ_G restricted to the non-negative integers.

(*ii*) Show that φ_G is well-defined.

(*iii*) Show that φ_G is a polynomial. What is its degree and what are the coefficients of the highest two powers?

(*iv*) Show that 0 is a root of φ_G and determine its multiplicity.

(*v*) Characterise the graphs G with $\varphi_G(x) = x^a(x-1)^{b-a}$ for given integers $b \geqslant a \geqslant 1$.

Exercise 2 [1 point]

Use König's Theorem to show that the complement of any bipartite graph is perfect.

Exercise 3 [1 point]

A graph is called a *comparability graph* if there exists a partial ordering of its vertex set such that two vertices are adjacent if and only if they are comparable. Show that every comparability graph is perfect.

Exercise 4 [1 point]

Show that the line graph $L(G)$ is perfect if and only if all odd cycles in G are triangles.