Fachbereich Mathematik | Universität Hamburg
Lecturer: Prof. Mathias Schacht | Fall 2020/21
Assistant: Dr. Christian Reiher | December 1st, 2020

Graph Theory 2

4th problem set

due December 8th, 10am

https://bit.ly/39w8D0F

Exercise 1 [1 point]

For a graph $G = (V, E)$ let $A(G) = (a_{uv})_{u,v \in V} \in \mathbb{R}^{|V| \times |V|}$ be its adjacency matrix over the reals with rows and columns indexed by the vertices, i.e., $a_{uv} = 1$ if $uv \in E$ and 0 otherwise. In particular, $A(G)$ is a symmetric square matrix with zeros on its diagonal. Owing to the symmetry, all its eigenvalues are real, which we denote by $\lambda_1(G) \geqslant \lambda_2(G) \geqslant \cdots \geqslant \lambda_{|V|}(G)$. Moreover, for every integer $k \geqslant 0$ the eigenvalues of the k-th power A_G^k are $\lambda_1^k(G), \lambda_2^k(G), \ldots, \lambda_{|V|}^k(G)$.

Find a graph theoretic interpretation of $\sum_{i=1}^{|V|} \lambda_i^k(G)$ for every $k \geqslant 0$ and prove it.

Exercise 2 [1 point]

(i) Find a 2-connected (multi)graph that has two non-isomorphic plane duals.

(ii) Show that the plane dual of a 3-connected graph is also 3-connected.

Exercise 3 [1 point]

Let G^* be an abstract dual of G, and let $e = e^*$ be an edge. Show that

(i) G^*/e^* is an abstract dual of $G - e$.

(ii) $G^* - e^*$ is an abstract dual of G/e.

Exercise 4 [1 point]

Prove the following statements:

(i) Every orientation of a bipartite graph has a kernel.

(ii) Every bipartite planar graph is 3-list-colourable.

(iii) Show that $K_{2,4}$ is not 2-list-colourable.