Fachbereich Mathematik
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Universität Hamburg
Fall 2020/21
November 24th, 2020

Graph Theory 2

3rd problem set

due December 1st, 10am

https://bit.ly/33cQIIB

Exercise 1 [1 point]

Prove or disprove that in a connected graph the minimal edge sets containing an edge from every spanning tree are precisely its bonds.

Exercise 2 [1 point]

Prove or disprove that the cycles and the cuts in a graph together generate its edge space.

Exercise 3 [1 point]

Prove or disprove that a 2-connected plane graph is bipartite if and only if every face is bounded by an even cycle.

Exercise 4 [1 point]

Find an algebraic proof of Euler's formula for 2-connected plane graphs along the following lines: Define the *face space* $\mathcal{F}$ (over $\mathbb{F}_2$) of such a graph in analogy to its vertex space $\mathcal{V}$ and edge space $\mathcal{E}$. Define *boundary maps* $\mathcal{F} \longrightarrow \mathcal{E} \longrightarrow \mathcal{V}$ in the obvious way, specifying them first on single faces or edges (i.e., on the standard bases of $\mathcal{F}$ and $\mathcal{E}$) and then extending these maps linearly to all of $\mathcal{F}$ and $\mathcal{E}$. Determine the kernels and images of these homomorphisms, and derive Euler's formula from the dimensions of those subspaces of $\mathcal{F}$, $\mathcal{E}$, and $\mathcal{V}$.