Fachbereich Mathematik
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Universität Hamburg
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Graph Theory 2

1st problem set

due November 17th, 10am

https://bit.ly/36erv0U

Exercise 1 [1 point]

For a graph $G = (V, E)$ and $X \subseteq V$ let $N^2(X)$ be the set of vertices outside X having at least two neighbours in X, i.e.,

$$N^2(X) = \left\{v \in V \smallsetminus X : |N(v) \cap X| \geqslant 2\right\}.$$

Suppose $|N^2(X)| \geqslant |X|$ for every independent subset $X \subseteq V$ of size at least 2. Show that G contains a matching covering all but at most one vertex.

Exercise 2 [1 point]

A graph $G = (V, E)$ is *vertex transitive* if, for any two vertices v, $w \in V$, there is an automorphism φ of G with $\varphi(v) = w$. Show that every connected, vertex-transitive graph with an even number of vertices contains a perfect matching.

Exercise 3 [1 point]

A planar graph is *outerplanar* if there exists a drawing in which every vertex lies on the boundary of the same face. Show that the edge set of every outerplanar graph is the union of two forests.

Exercise 4 [1 point]

Show that every graph, which does not contain two vertex disjoint cycles, can be turned into a forest by removing at most three vertices.