Fachbereich Mathematik
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Universität Hamburg
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Graph Theory 2

Exercise Sheet 12

due on February 1, 1pm

http://bit.ly/2F76Hcu

Exercise 1 (§12.24) [1 point]
Show that for $n \geqslant 3$ the clique K_n, the cycle C_n, an arbitrary tree T_n of order n, and the $n \times n$ grid have tree-decompositions of widths $n-1$, 2, 1, and n, respectively. For K_n and C_n show that these values are best possible.

Exercise 2 (§12.25) [1 point]
Can the tree-width of a subdivision of a graph G be smaller than the tree-width of G? Can it be larger?

Exercise 3 (§12.26) [1 point]
Show that the tree-width of a finite graph is at least its minimum degree.

Exercise 4 (§12.31) [1 point]
A tree–decomposition whose tree is a path is a *path-decomposition*. The *path-width* of G is the least width of a path-decomposition of G. Show that trees have unbounded path-width, i.e., the path-width of trees can not be bounded by a universal constant independent of the tree under consideration.

Written Exercise (§12.28)
A graph is called *outerplanar* if it has a drawing in which every vertex lies on the boundary of the outer face. Show that the tree-width of outerplanar graphs can be bounded by some constant independent of the size of the graph.