Fachbereich Mathematik | Universität Hamburg
Lecturer: Prof. Mathias Schacht | Fall 2017/18
Assistant: Oliver Ebsen | January 18th, 2018

Graph Theory 2

Exercise Sheet 11

due on January 25, 1pm

http://bit.ly/2DJmb79

Exercise 1 (§9.5) [1 point]

Construct a graph on $\mathbb{R}$ that has neither a complete nor an edgeless induced subgraph on $|\mathbb{R}| = 2^{\aleph_0}$ vertices. (So Ramsey's theorem does not extend to uncountable sets.)

Hint: Choose a well-ordering of $\mathbb{R}$, and compare it with the natural ordering. For countability arguments, use the rationals.

Exercise 2 (§10.6) [1 point]

A graph $G = (V, E)$ is *t-tough* for some real number $t > 0$, if for every nonempty set $S \subseteq V$ the graph $G - S$ has at most $\lceil |S|/t \rceil$ components.

(*i*) Show that every Hamiltonian graph is 1-tough.

(*ii*) Find a graph that is 1-tough but not Hamiltonian.

Exercise 3 (§10.10) [1 point]

The *r-th power* G^r of a graph G is defined on the same vertex set with vertices being adjacent, if their distance in G is at most r. Prove that the square G^2 of a k-connected graph G is k-tough.

Exercise 4 (§10.14) [1 point]

Show by induction on $|V(G)|$ (possibly with strengthened induction assumption) that the third power G^3 of any connected graph G contains a Hamiltonian cycle.

Written Exercise

Deduce the following asymmetric, multicolour version from the symmetric 2-colour case of the induced Ramsey theorem for graphs (Theorem 9.3.1): *For every integer $q \geqslant 1$ and graphs $F_1, \dots, F_q$ there exists a graph G such that for every q-colouring $\varphi\colon E(G) \to [q]$ there is some $i \in [q]$ and an induced copy of F_i in G with all edges in $\varphi^{-1}(i)$.*