Fachbereich Mathematik
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Universität Hamburg
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Graph Theory 2

Exercise Sheet 10

due on January 18, 1pm

http://bit.ly/2AQ0kHU

Exercise 1 [1 point]

For graphs G and H and an integer $q \geqslant 1$ we use the *arrow notation* $G \longrightarrow (H)_q$ of Erdős, Hajnal, and Rado as a shorthand for the validity of the statement that every colouring $\varphi\colon E(G) \to [q]$ yields a monochromatic copy of H. Prove or disprove the following statements.

(*i*) For every $q \geqslant 1$ and every path P there is a path Q such that $Q \longrightarrow (P)_q$.

(*ii*) For every $q \geqslant 1$ and every star S there is a star T such that $T \longrightarrow (S)_q$.

(*iii*) For every $q \geqslant 1$ and every tree T there is a tree Z such that $Z \longrightarrow (T)_q$.

Exercise 2 (§9.9) [1 point]

Show that for every integer $q \in \mathbb{N}$ there is an $n \in \mathbb{N}$ such that, for every partition of $[n]$ into q sets, at least one of the subsets contains numbers x, y, z such that $x + y = z$.

Hint: Ramsey's theorem for triangles.

Exercise 3 (§9.13) [1 point]

Let m, $n \in \mathbb{N}$, and assume that $m-1$ divides $n-1$. Show that for every tree T with m vertices the Ramsey number $R(T, K_{1,n})$ equals $m+n-1$, i.e., $m+n-1$ is the smallest integer N, such that every two-colouring of the edges of K_N with colours red and blue either yields a red copy of T or a blue copy of $K_{1,n}$.

Exercise 4 (§9.18) [1 point]

Show that, given any two graphs H_1 and H_2, there exists a graph $G = G(H_1, H_2)$ such that, for every *vertex colouring* $\varphi\colon V(G) \to [2]$ of G with colours 1 and 2, there is either an induced copy of H_1 coloured 1 or an induced copy of H_2 coloured 2 in G. Does the proof extend for more than two colours?

Written Exercise (§9.14)

We denote by $R(F; q)$ the smallest inter N such that every q-colouring of the edges of K_N yields a monochromatic copy of F. Prove $2^q < R(K_3; q) \leqslant 3q!$ for every integer $q \geqslant 1$.