Fachbereich Mathematik | Universität Hamburg
Lecturer: Prof. Mathias Schacht | Fall 2017/18
Assistant: Oliver Ebsen | November 30th, 2017

Graph Theory 2

Exercise Sheet 7

due on December 7, 1pm

http://bit.ly/2zEfwYQ

Exercise 1 (§6.16) [1 point]

Show that every graph with a Hamiltonian cycle has a 4-flow.

Exercise 2 (§6.17) [1 point]

A family of (not necessarily distinct) subgraphs of a graph G is called a *double cover* of G if every edge of G lies on exactly two of these subgraphs. The *cycle double cover conjecture* asserts that every bridgeless multigraph admits a double cover by cycles. Prove the conjecture for graphs with a 4-flow.

Exercise 3 (§6.19) [1 point]

Find bridgeless graphs G and $H = G - e$ such that $2 < \varphi(G) < \varphi(H)$.

Exercise 4 (§6.21) [1 point]

Prove that a plane triangulation is 3-colourable if and only if all its vertices have even degree.

Written Exercise (§6.24)

Show that a graph $G = (V, E)$ has a k-flow if and only if it has an orientation D that directs, for every $X \subseteq V$, at least $1/k$ of the edges in $E(X, \overline{X})$ from X towards $\overline{X}$.