Fachbereich Mathematik — Universität Hamburg
Lecturer: Prof. Mathias Schacht — Fall 2017/18
Assistant: Oliver Ebsen — November 23rd, 2017

Graph Theory 2

Exercise Sheet 6

due on November 30, 1pm

http://bit.ly/2jixTMm

Exercise 1 (§6.6) [1 Punkt]

Let H be an abelian group, $G = (V, E)$ be a connected graph, T be a spanning tree, and f be a map from the orientations of the edges in $E \setminus E(T)$ to H that satisfies (F1). Show that f extends uniquely to a circulation on G with values in H.

Exercise 2 (§6.7) [1 Punkt]

Continuing with the setup of Exercise 1, let $\mathcal{V}_H$ be the group of all maps $V \to H$ and let $\mathcal{E}_H$ be the group of all maps $\vec{E} \to H$ satisfying (F1), both with pointwise addition. Every $\varphi \in \mathcal{V}_H$ defines a $\psi \in \mathcal{E}_H$ by $\psi(e, x, y) := \varphi(y) - \varphi(x)$.

(i) Show that these ψ form a subgroup $\mathcal{B}_H$ of $\mathcal{E}_H$ with

$$\mathcal{B}_H = \left\{\psi \in \mathcal{E}_H \colon \psi(\vec{C}) = 0 \text{ for every oriented cycle } C \subseteq G\right\},$$

where $\psi(\vec{C}) = \sum\{\psi(\vec{e}) \colon \vec{e} \in \vec{C}\}$.

(ii) Show that every map $\vec{E}(T) \to H$ satisfying (F1) extends uniquely to a map in $\mathcal{B}_H$.

Exercise 3 (§6.8) [1 Punkt]

Continuing with the setup of Exercises 1 and 2, let $\mathcal{C}_H$ denote the group of all circulations on G with values in H.

(i) Show that the quotient group $\mathcal{E}_H/\mathcal{B}_H$ is isomorphic to $\mathcal{C}_H$.

(ii) Show that the quotient group $\mathcal{E}_H/\mathcal{C}_H$ is isomorphic to $\mathcal{B}_H$.

Exercise 4 (§6.12) [1 Punkt]

Show (directly, without using Theorem 6.6.1) that every bridgeless multigraph G has a k-flow for some $k \in \mathbb{N}$.

Written Exercise (§6.5)

View the group of circulations on a graph with values in $\mathbb{Z}/2\mathbb{Z}$ as a vector space over $\mathbb{Z}/2\mathbb{Z}$. Find a space to which it is isomorphic, and write down an explicit isomorphism.