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Ramsey Theory

Exercise Sheet 4

due date: July 7th, 2014 - 12:01pm
`http://ow.ly/ywkJp`

Exercise 1
Formulate an extension of the Hales-Jewett theorem in the same spirit as Szemerédi's theorem generalizes van der Waerden's theorem and prove it for an alphabet of size two.

Exercise 2
For a finite set X and a family $\mathcal{F} \subset 2^X$ of subsets of X we say that $\mathcal{F}$ contains a *Boolean cube of dimension* $d \in \mathbb{N}$, if there are mutually disjoint subsets $A, B_1, \dots, B_d \subseteq X$ such that $A \cup \bigcup_{i \in I} B_i \in \mathcal{F}$ for every $I \subseteq [d]$.
Prove that for any $d \in \mathbb{N}$ and any finite partition of 2^X for sufficiently large X there exists one partition class containing the d-dimensional Boolean cube. Can it be ensured that such a the d-dimensional cube is always contained in the largest partition class?

Exercise 3

(*i*) Prove the following quantitative version of Ramsey's theorem: For every $\ell > k \geq 2$ and $r \geq 2$ there exist $\alpha > 0$ and n_0 such that for every $n \geq n_0$ every r-colouring of the hyperedges of the complete k-uniform hypergraph on n vertices yields at least $\alpha\binom{n}{\ell}$ monochromatic copies of $K_\ell^{(k)}$. What can you say about the optimal value of α as a function of ℓ, k, and r.

(*ii*) In view of (*i*), formulate an appropriate strengthening of van der Waerden's theorem. Does it hold?

Exercise 4
Find a colouring of the Hales-Jewett cube, which shows that a quantitative extension of the Hales-Jewett theorem (in the sense of Exercise 3) fails to be true.