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Ramsey Theory

Exercise Sheet 4

due date: June 16th, 2014 - 12:01pm
`http://bit.ly/1xzkxvX`

Exercise 1

(*i*) Let $R = R(n,n)$ be the two-colour Ramsey number for K_n and let $H \subsetneq K_R$. Show that there exists a two-colouring of the edge set of H with no monochromatic K_n.

(*ii*) For a graph F and an integer r we denote by $\widehat{R}(F;r)$ the smallest integer m such that there exists a graph H with m edges such that every r-colouring of the edge set of H yields a monochromatic copy of F. Show that $\widehat{R}(K_n;r) = \binom{R(n;r)}{2}$.

Exercise 2
Show that the density version of Hilbert's cube lemma is asymptotically optimal in the sense that there are subsets of $[n]$ of density $\alpha > 0$, with the largest cube of dimension $O(\log \log n)$.

Exercise 3
A well-known result of Ajtai and Szemerédi asserts that for every $\delta > 0$ there exists n_0 such that every subset $A \subseteq [n]^2$ with $n \geq n_0$ and $|A| \geq \delta n^2$ contains a *corner*, i.e., three points of the form (x,y), $(x+d,y)$ and $(x,y+d)$ for some $d \neq 0$. Deduce $r_3(n) = o(n)$ from the Ajtai-Szemerédi theorem.

Exercise 4
Show that every red-blue-colouring of the edge set of K_n yields a red cycles and a blue cycle, which share at most one vertex and cover all vertices. Here are K_1 and K_2 considered to be cycles.

Hint: Consider first Hamiltonian cycles consisting of two monochromatic paths.