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Ramsey Theory

Exercise Sheet 3

due date: May 19th, 2014 - 12:01pm
`http://ow.ly/wJPdV`

Exercise 1
Deduce from Schur's theorem, that Fermat's last theorem fails in modular arithmetic, i.e., that for every $m \in \mathbb{N}$ there exists p_0 such that for every prime $p > p_0$ there exist integers x, y, and z satisfying

$$x^m + y^m \equiv z^m \mod p$$

and p does not divide x, y, and z.

Exercise 2
Prove the following common generalization of Schur's theorem and van der Waerden's theorem: For all integers k, $r \geq 2$ there exists n_0 such that for every $n \geq n_0$ and every partition $E_1 \dot\cup \ldots \dot\cup E_r = [n]$ there exists a $j \in [r]$ such that E_j contains an arithmetic progression of length k with common difference $d > 0$ and $d \in E_j$.

Hint: First consider the case $r = 2$.

Exercise 3
Prove a lower bound for the van der Waerden number $W(k; r)$, i.e., for the $n_0 = n_0(k, r)$ in Theorem 3.1.

Exercise 4
Obtain an upper bound on $W(k; 2)$ from the combinatorial proof of van der Waerden's theorem.