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Ramsey Theory

Exercise Sheet 2

due date: May 5th, 2014 - 12:01pm
`http://ow.ly/vVnIV`

Exercise 1
We consider a version of Ramsey's theorem for relatively large subsets of $\mathbb{N}$. We say a finite subset $A \subset \mathbb{N}$ is *relatively large* if $\min A := \min_{a\in A} a \leq |A|$.

Show that the finite version of Ramsey's theorem (Theorem 2.1) holds with the additional property that Y is relatively large, i.e., show that for all integers $r,\ k,\ \ell \geq 1$ there exists an $n \in \mathbb{N}$ such that for every partition $E_1 \dot\cup \dots \dot\cup E_r = \binom{[n]}{k}$ there exists a relatively large subset $Y \subseteq [n]$ with $|Y| \geq \ell$ and $\binom{Y}{k} \subseteq E_j$ for some $j \in [r]$.

Hint: Compactification!

Exercise 2
We consider a version of Ramsey's theorem for non-uniform hypergraphs. For a set X we denote by $\binom{X}{<\aleph_0} = \bigcup_{n\in\mathbb{N}} \binom{X}{n}$ be the set of all finite subsets of X. Show that for all integers $r,\ k \geq 1$, and every infinite set X the following holds. For any partition $E_1 \dot\cup \dots \dot\cup E_r = \binom{X}{<\aleph_0}$ there exists an infinite subset $Y \subseteq X$ such that for every $i \in [k]$ there exists an index $j_i \in [r]$ with $\binom{Y}{i} \subseteq E_{j_i}$.

Exercise 3
Prove or disprove the following extensions of the non-uniform Ramsey theorem (see Exercise 2):

(*a*) The conclusion holds with $j_1 = \dots = j_k$.

(*b*) The conclusion holds for $k = \aleph_0$, i.e., there exists an infinite set Y with the property that for every $n \in \mathbb{N}$ there exists a j_n such that $\binom{Y}{n} \subseteq E_{j_n}$.

Hint: Let $X = \mathbb{N}$ and consider relatively large finite subsets (see Exercise 1).

Exercise 4
Show that a version of Ramsey's theorem with infinite edges fails, i.e., show that for every infinite set X there exists a partition $E_1 \dot\cup E_2 = \binom{X}{\aleph_0}$ of the countably infinite subsets of X such that every infinite subset $Y \subseteq X$ contains countably infinite subsets from E_1 and from E_2.

Hint: Consider a well-ordering $\prec$ of $\binom{X}{\aleph_0}$ and let E_1 be the countably infinite sets that contain a smaller subset.