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Ramsey Theory

Exercise Sheet 1

due date: April 14th, 2014 - 12:01pm
`http://ow.ly/vB5sS`

Exercise 1
Deduce Theorem 1.1 for general r from the statement for $r = 2$. Does the same implication hold for the other Ramsey-type results in Section 1.1?

Exercise 2
Deduce Hilbert's cube lemma (Theorem 1.2), Schur's theorem (Theorem 1.3), and van der Waerden's theorem (Theorem 1.4) from Rado's theorem (Theorem 1.5).

Hint: Exclude "degenerate" solutions, which are not allowed in Theorems 1.2 and 1.4.

Exercise 3

(*a*) Does the Graham-Leeb-Rothschild theorem (Theorem 1.8 and 1.9) hold, when the finite field $\mathbb{F}$ is replaced by an infinite field, say $\mathbb{R}$?

(*b*) Formulate a density version of the finite version of Ramsey's theorem. Does it hold? Can you formulate one that is true?

(*c*) Does van der Waerden's theorem hold for an infinite number of colours? Does it hold for an unbounded number of colours?

Exercise 4
State an example of an equation that does not fulfill Rado's condition and give an explicit colouring of $\mathbb{N}$ with no monochromatic solution for this equation.