2d Spin TFTs from defect TFTs

Sebastian Novak

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Outline

Motivation

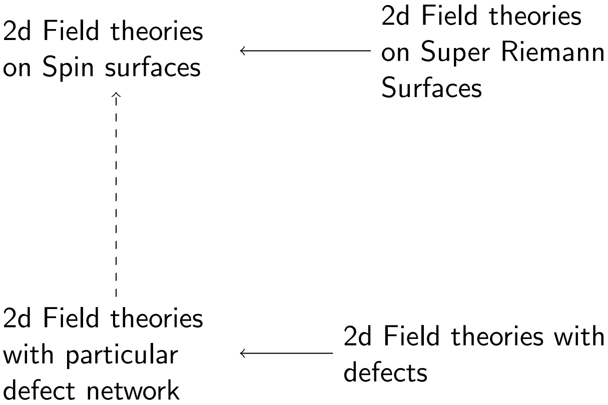
The state sum construction for 2d TFTs
- Algebraic data
- Geometric construction
- Independence of Triangulation and Extra data

The state sum construction for 2d Spin TFTs
- Spin structures on surfaces
- Geometric construction
- New algebraic data

2d Spin Field Theory via 2d Field Theory with Defects

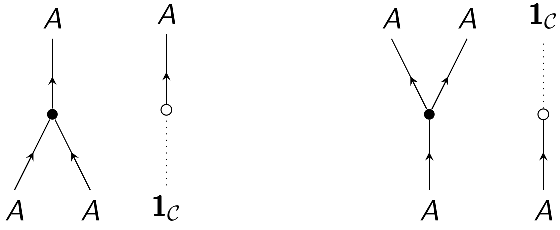
Outlook

Motivation


2d Field theories on Spin surfaces
2d Field theories on Super Riemann Surfaces
2d Field theories with particular defect network
2d Field theories with defects

algebraic data I

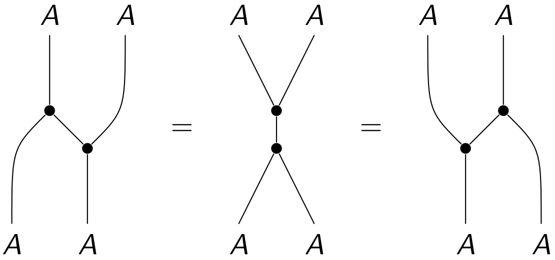
$\mathcal{C}$: symmetric monoidal category (i.e. $\mathcal{C} = \mathbf{Vect}_{\mathbb{C}}$ or $\mathcal{C} = \mathbf{SVect}_{\mathbb{C}}$).
$(A, \mu, \eta, \Delta, \varepsilon)$: symmetric Δ-separable Frobenius algebra in $\mathcal{C}$.



μ η Δ ε

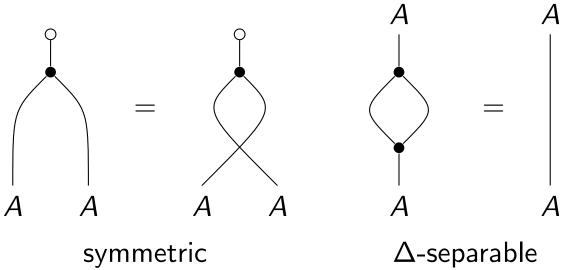
associative unital algebra coassociative counital coalgebra

algebraic data II: Frobenius algebra relations



Frobenius relation.

algebraic data III



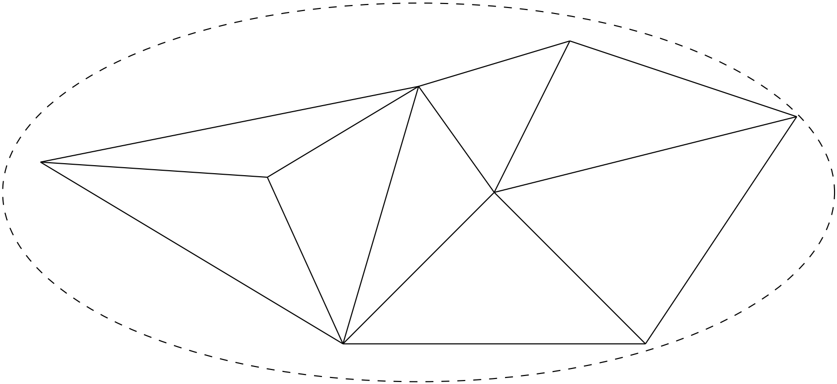
Geometric construction: Overview

State sum construction for 2d Topological Field Theory from this algebraic data:
[Bachas Petropoulos '92][Fukuma Hosono Kawai '92][Lauda Pfeiffer '06]

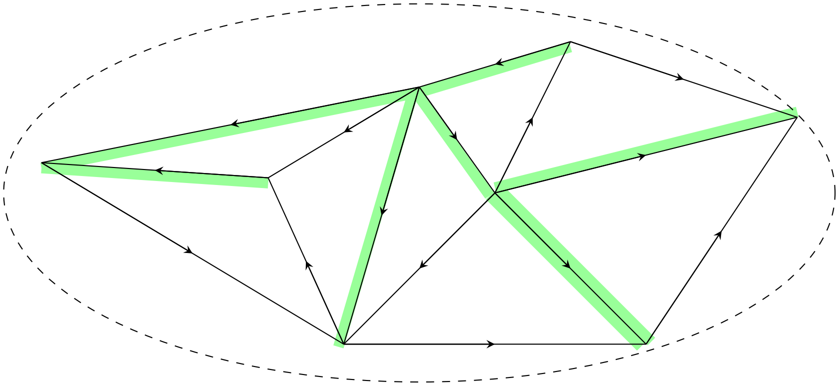
1. Triangulate (oriented) surface Σ.
2. Use simplicial structure to construct a morphism $Z(\Sigma)$ in $\mathcal{C}$.
3. Show independence of the triangulation by using Pachner moves.

Focus now on closed surfaces, then $Z(\Sigma) : \mathbf{1}_{\mathcal{C}} \to \mathbf{1}_{\mathcal{C}}$.
For $\mathcal{C} = \mathbf{Vect}_{\mathbb{C}}$: $Z(\Sigma) \in \mathbb{C}^{\times}$.

Step 1: Triangulate Surface

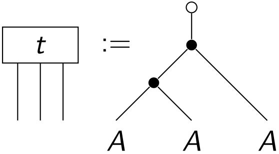


Step 2: Extra data

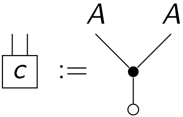


- Pick direction for each edge.
- Mark edge for each triangle.

Two morphisms in $\mathcal{C}$

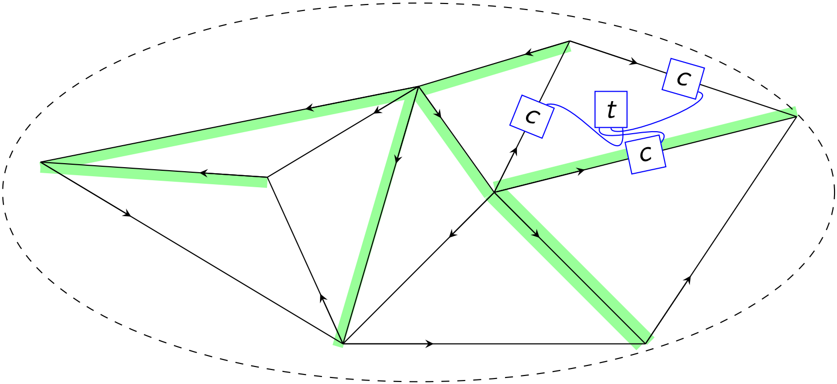


Trilinear map



Copairing

Step 3:Construction of morphism

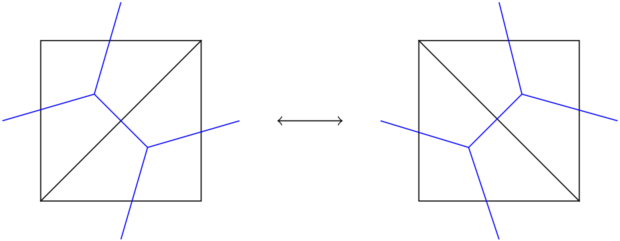


Independence of Extra data

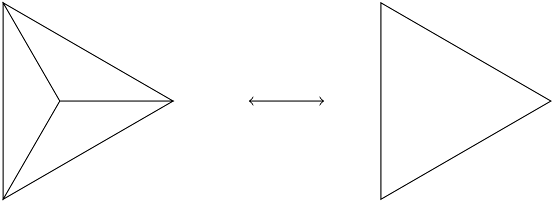
- Edge direction: Symmetry of (co-)pairing.
- Marked edge: Cyclicity of trilinear map t. Follows from symmetry.

Independence of Triangulation

- Invariance of morphism under 2d Pachner moves



By Frobenius property.

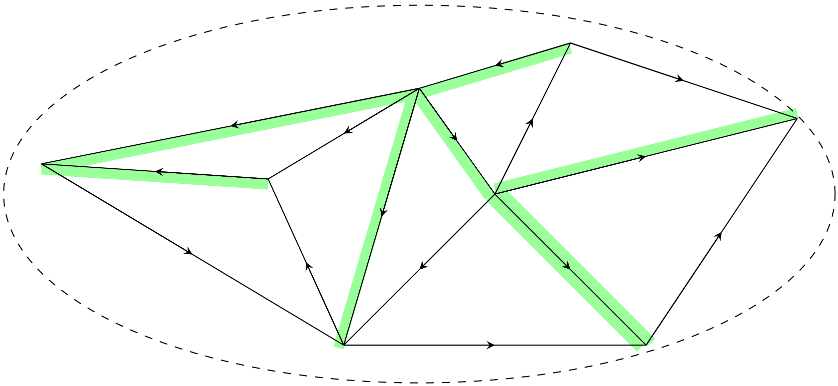


By Δ-separability.

State sum construction for spin surfaces.

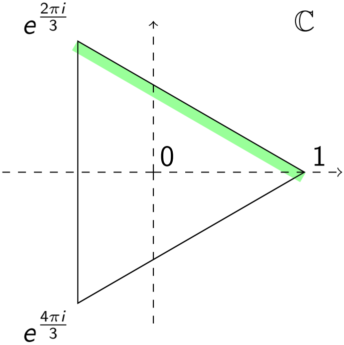
- Spin structure: Double cover of bundle of oriented frames (nontrivial on each fiber).
- No metric.
- Spin TFT can depend only on isomorphism class of spin surface.

Step 1: Triangulation + Extra data



- Smooth triangulation: $\varphi : |\mathbf{C}| \to \Sigma$ ($\mathbf{C}$: simplicial complex). (*Choice 1*)
- Same extra data on triangulation as before. (*Choice 2*)

Step 2: Computation of spin indices

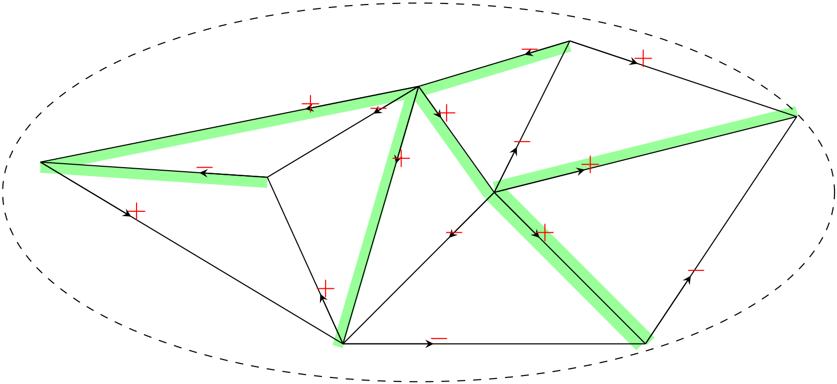


standard triangle Δ.

Step 2: Computation of spin indices

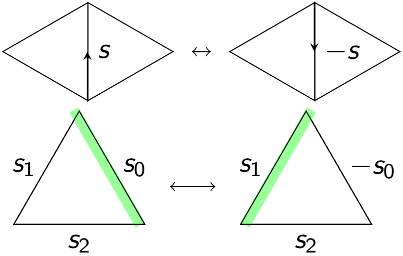
- Triangulation + Extra data $\Rightarrow$ char. map $\chi_\sigma : \Delta \to \Sigma$ for each triangle $\sigma \in \mathbf{C}$.
- Pick spin lifts $\tilde{\chi}_\sigma$ for char. maps. (*Choice 3*).
- Transition functions $((\chi_\sigma)^{-1} \circ \chi'_\sigma)$ are "rotations".
- Pick spin lifts for rotations $\mathbb{C} \to \mathbb{C}$.
- Comparison of spin transition functions $((\tilde{\chi}_\sigma)^{-1} \circ \tilde{\chi}'_\sigma)$ with chosen lifts of rotations:
  $\Rightarrow$ spin index $s_e = \pm 1$ for each edge e.
- The spin structure can be reconstructed (up to isomorphism) from these indices.

Triangulation with spin indices

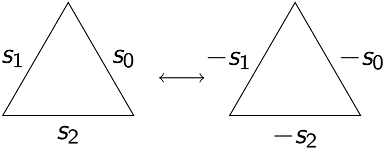


Index dependence on choices

- Choice 2: Extra data on triangulation.

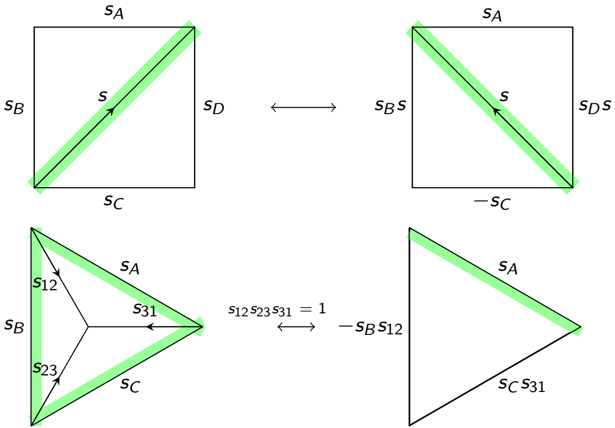


- Choice 3: Spin lifts for characteristic maps.

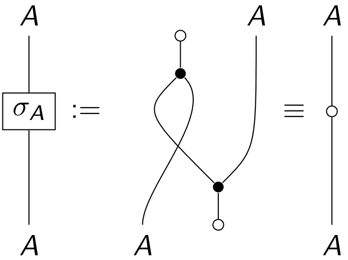


Index dependence on extra data II

- Choice 1: Triangulation.



Nakayama automorphism



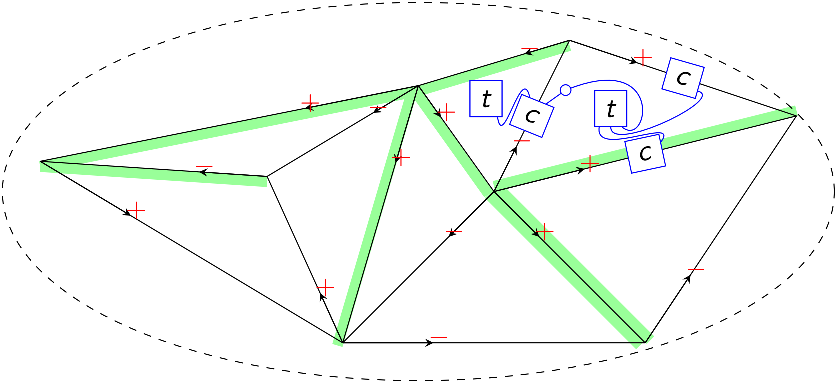
- Always Frobenius algebra automorphism.
- Natural in the Frobenius algebra A.
- If the Frobenius algebra is symmetric, then $\sigma = \mathrm{id}_A$.

new algebraic data

$(A, \mu, \eta, \Delta, \varepsilon)$: Δ-separable Frobenius algebra with $(\sigma \circ \sigma) = \mathrm{id}_A$.
Examples:

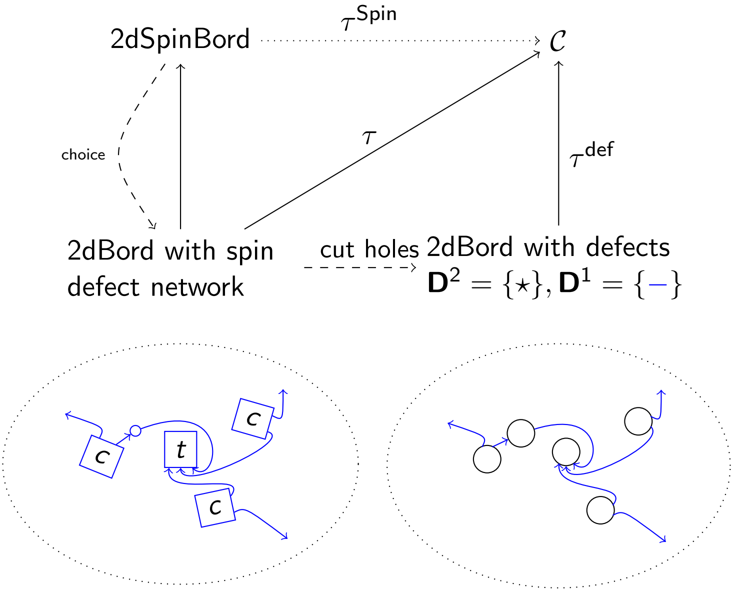
- Symmetric Frobenius algebras.
- Twist of counit of a symmetric Frobenius algebra with an element whose inner automorphism is an involution.
  I.e. $t \in A$ such an element, $\varepsilon'(x) = \varepsilon(t \cdot x)$ for all $x \in A$.

Step 3:Construction of morphism



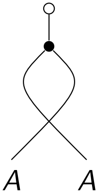
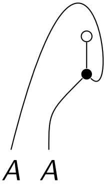
Same construction as before, but Nakayama automorphism is inserted once for each edge with a minus sign.
Algebraic data ensures invariance under moves, thus invariance under choices 1-3.

2d Spin TFT via 2d TFT with Defects



2d Spin TFT via 2d TFT with Defects

- 2d def. TFT τ^{def} $\Rightarrow$ pivotal monoidal category of defects $\mathcal{D}_{\tau^{\text{def}}}$.
  [Davydov Kong Runkel '11][Carqueville Runkel '12]

Replace  with  .

- Pick Δ-separable Frobenius algebra with $\sigma^2 = \text{id}$ in $\mathcal{D}_{\tau^{\text{def}}}$.
  $\Rightarrow$ Spin TFT by use of choices to get the defect network.

Outlook

- Rational 2dCFTs
- Relation to other descriptions of 2dSpinTFT
- Other geometric structures on surfaces

Thank You!