Strings, symmetries and representations

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Abstract.   Several aspects of symmetries in string theory are reviewed. We discuss the rôle of symmetries both of the string world sheet and of the target space. We also show how to obtain string scattering amplitudes with the help of structures familiar from the representation theory of quantum groups.

1. Strings and conformal field theory

Symmetries of various types, and consequently representation theoretic tools, play an important rôle in string theory and conformal field theory. The present contribution aims at reviewing some of their aspects, the choice of topics being influenced by our personal taste. After a brief overview of string theory and conformal field theory, we first discuss orbifolds and duality symmetries. We then turn to D-branes and theories of open strings, which we investigate in the final section using Frobenius algebras in representation categories.

A minimalistic point of view on (perturbative) string theory is to regard it as a perturbative quantization of a field theory, with the perturbation expansion being organized not in terms of graphs, i.e. one-dimensional objects, but rather in terms of surfaces. What makes this perturbation expansion particularly interesting is that it even covers the quantization of theories that include a gravitational sector.

The configuration space of a classical string is given by the embeddings of its two-dimensional world sheet Σ, with local coordinates τ and σ, into a target space M with coordinates X^μ. Both Σ and M are supposed to be endowed with a metric and possibly other background fields. It is convenient to change the perspective and regard the target space coordinate X^μ, via the embedding of Σ into M, as a function of the world sheet coordinates τ, σ, and thus as a classical field $X^\mu(\tau, \sigma)$ on Σ. This classical field theory is governed by an action S, the sigma-model action, which depends on the embedding X and on the metric h on the world sheet. S must in particular be invariant under local rescalings of the metric h of Σ

$$S[\mathrm{e}^{\Phi(\sigma,\tau)}h, X] = S[h, X] \tag{1}$$

so that we deal with a classical 2D conformal field theory (CFT). Such theories carry an action of an infinite-dimensional symmetry algebra, the Witt algebra, respectively (after quantization) of its central extension, the Virasoro algebra. There can also be further symmetries, so that the Virasoro algebra is generically only a subalgebra of the so-called *chiral algebra*, which is provided by all holomorphic fields in the theory (the field affiliated with the Virasoro algebra is the stress-energy tensor T). In simple cases, like a flat background

$M = \mathbb{R}^{p,q}$, the solutions of the classical field equations split into left moving and right moving modes,

$$X^\mu(\sigma, \tau) = X_L^\mu(\sigma{+}\tau) + X_R^\mu(\sigma{-}\tau). \tag{2}$$

When the classical field theory can be quantized, one obtains a quantization of the space-time coordinates as fields in a two-dimensional theory. As a result there is a rich interplay between world sheet physics and space-time physics. For instance, the vanishing of the β-functions in the two-dimensional field theory implies field equations of a theory on target space that include a gravitational sector. To describe string theory, the BRST procedure must be applied to the conformal symmetry. Nilpotency of the BRST charge restricts the target space M, in the case of superstrings and of a flat metric on M, to 9+1 space-time dimensions. To give another example, modular invariance of the torus partition function

$$Z(\beta) = \mathrm{tr}_{\mathcal{H}_{\mathrm{string}}} e^{-\beta H_{\mathrm{string}}} \tag{3}$$

of the world sheet theory, i.e. invariance of $Z(\beta)$ under transformations

$$\mathrm{i}\beta \;\mapsto\; \frac{a\,\mathrm{i}\beta + b}{c\,\mathrm{i}\beta + d} \tag{4}$$

with $a, b, c, d \in \mathbb{Z}$, $ad - bc = 1$, implies absence of anomalies [28]. One should be aware, though, that the converse is not true – it is easy to write down anomaly-free spectra not coming from a modular invariant partition function. Thus, at least as closed strings are concerned, string theory is a strictly more selective theoretical framework than particle field theory.

Bosonic string theory has a serious drawback: its spectrum contains tachyons, i.e. states of negative mass squared. This problem can be circumvented by enlarging the symmetries of the world sheet to a Lie *super*algebra containing the super-Virasoro algebra. Then among the holomorphic fields one finds, along with the stress-energy tensor T, its superpartner, the supercurrent G. Correspondingly, the world sheet must now carry the structure of a super-Riemann surface, rather than a Riemann surface. (In practice, one often works in a bosonic setting, employing the equivalence between super-Riemann surfaces and Riemann surfaces with a spin structure. Special care is then required, in particular when studying such theories on surfaces with boundaries [16].) The extension to world sheet supersymmetry can be implemented either for both left moving and right moving degrees of freedom, leading to superstring theories of type II, or for one chirality only, leading to heterotic strings.

If *space-time* supersymmetry is imposed as an additional requirement, say for 'phenomenological' reasons in compactifications to four space-time dimensions, then a further extension of the world sheet symmetry is required. One needs $N = 2$ supersymmetry on the world sheet (of which an $N = 1$ subalgebra is gauged), which means the presence of two supercurrents $G^{(\pm)}$ as well as an abelian current J. In addition a projection, the so-called GSO projection, must be imposed. This can be done in two different ways, depending on a relative sign for left movers and right movers, which leads to superstring theories of type IIA and IIB, respectively.

As mentioned above, for superstrings BRST invariance requires space-time to be 9+1-dimensional. But the space-time we experience has only 3+1 dimensions. Therefore one considers targets M for which six dimensions are compactified, i.e., in the simplest case,

$$M = \mathbb{R}^{3,1} \times M_6 \tag{5}$$

with some compact Euclidean six-dimensional internal space M_6.

There are two major approaches for arriving at such a string background:

■ A geometric setting, in which one studies a sigma model with target space of the form (5). Space-time supersymmetry then requires the compact six-dimensional space M_6 to be a Calabi-Yau manifold, i.e. to admit a covariantly constant spinor.

■ An entirely different approach describes the internal space through exactly solvable CFTs built from infinite-dimensional symmetry algebras, in particular affine Lie algebras. (Affine Lie algebras also provide the gauge sector in heterotic theories.) This purely algebraic construction allows one to investigate string theory in the regime of strong curvature of the target space in which geometric methods break down. More specifically, for every homogeneous space G/H, the so-called coset construction supplies an exactly solvable CFT model. It can be proven [23, 30] that such a coset model has $N = 2$ superconformal symmetry if and only if G/H is Kähler. These models, called Kazama-Suzuki models (including $N = 2$ minimal models), are the building blocks for the Gepner construction of string vacua.

2. Orbifolds

It is a general experience that physical systems with symmetries admit reduction procedures which can result in interesting new systems. String theory is no exception; here reduction appears in the form of orbifold methods, which constitute a powerful tool for obtaining new string backgrounds. The orbifold construction has an algebraic and a geometric aspect, too.

On the algebraic side one considers the situation that the chiral algebra $\mathcal{A}$ has a non-trivial group G of automorphisms. The fixed point subalgebra $\mathcal{A}^G$ of $\mathcal{A}$ is again a chiral algebra – the chiral algebra of the *orbifold theory* [5]. Results similar to the theory of dual pairs (cf. [19] for a review) give a good control on the representations of $\mathcal{A}^G$ and thus on the superselection sectors of the (chiral) orbifold theory. The chiral data of orbifold theories have been worked out explicitly for large classes of examples, like orbifolds of affine Lie algebras by inner [22] and outer ([4], compare also [15]) automorphisms and permutation orbifolds [2]. The formalization of chiral algebras as vertex algebras allows one to prove many statements about the representation theory of orbifolds. There is in particular a Galois theory for chiral algebras [7].

In the geometric setting [6], one considers the action of a group G on a target space M by isometries. (This action must also preserve other background fields than the metric.) In case the action is not free, this gives rise to a singular space $M//G$. However, as a consequence of including so-called twisted sectors, which is necessary to achieve modular invariance, string propagation in such a background is still well-defined, i.e. the CFT does not feel the singularity. It is worth emphasizing that for those models which possess both a geometric and an algebraic description, the geometric orbifold does not always yield the same theory as an algebraic orbifold. For example, when M is a compact Lie group and G a subgroup of its center, the geometric orbifold construction can lead to an enhancement, rather than a reduction, of the chiral algebra. (This happens e.g. for $M = \mathrm{SU}(2)$ and $G = \mathbb{Z}_2$, i.e. for the WZW model based on the group $\mathrm{SO}(3)$. If the level is divisible by four, then the corresponding modular invariant is of extension type.)

As a technical aside, we mention that the computation of string amplitudes that include twist fields is based on the theory of Riemann surfaces with holomorphic symmetries [18].

3. Duality symmetries

Another task that symmetries can fulfill is to relate different backgrounds. In the simplest case of a free background, along with the combination (2) obviously also the function

$$\tilde{X}^\mu(\sigma, \tau) = X_L^\mu(\sigma + \tau) - X_R^\mu(\sigma - \tau) \tag{6}$$

is a classical solution. A more complete analysis shows that this duality continues to be realized in the quantized theory; it connects the theory of a free boson compactified on a circle of radius R with the same theory compactified on a circle of radius $2/R$.

This operation is known as *T-duality*. It typically connects two *different* backgrounds; self-dual theories possess an enhanced symmetry. For example, at radius $R^2 = 2$ the free boson on a circle acquires a non-abelian current algebra of type $\widehat{\mathfrak{su}}(2)$ at level 1 as an additional symmetry. This is a generic feature of duality symmetries. It already appears in the Kramers-Wannier duality of lattice systems that relates the high- and low-temperature regime; the self-dual theory is at the critical point (the reader may guess herself what the enhanced symmetry is in this example). A direct lesson from T-duality for string theory is that geometric interpretations of string theory backgrounds typically require to make arbitrary choices – e.g. in the case of a compactified free boson none of the two possible interpretations, i.e. either using the radius R or using $2/R$, is preferred.

T-duality possesses the following important generalization. The $N = 2$ superconformal algebra has an automorphism of order two, acting as

$$\omega(J) = -J \,, \qquad \omega(G^{(\pm)}) = G^{(\mp)} \,, \qquad \omega(T) = T \,, \tag{7}$$

which just like T-duality flips the sign of a chiral abelian current, and as a consequence can also be used to relate different string backgrounds. In the geometric setting, it relates different Calabi-Yau manifolds. This relationship, called *mirror symmetry*, has turned out to be both mathematically deep and technically extremely useful (for a review, see e.g. [17]).

The dualities discussed so far are so-called perturbative dualities – they relate terms at the same order in string perturbation theory. ‡ In contrast, dualities like the Olive-Montonen duality are non-perturbative. Such dualities have found many generalizations in string theory in the last eight years. In particular, so-called S-dualities act also on the coupling constant and thereby allow us to relate different backgrounds in the regime of strong and weak coupling. For example, it has been found that the type IIA superstring on K3 surfaces is connected via such a duality with the heterotic string on a four-dimensional torus [20].

4. D-branes and open strings

So far we have restricted our attention to symmetries acting either on the target space or on the chiral algebra. There are also symmetries that act on the world sheet as well. These have been termed *orientifolds*; they result in yet another type of string theories, of type I, in which the perturbation theory is organized in terms of orientable *and* unorientable surfaces that are also allowed to have boundaries. In these theories the torus amplitude $Z(\tau)$, constituting the partition function of bulk fields, is projected by a contribution given by the amplitude $K(t)$ of the CFT on a Klein bottle, which leads to a total partition function $\frac{1}{2}(Z + K)$. In terms of the characters $\chi_i(\tau)$ of the chiral CFT, the torus amplitude reads

$$Z(\tau) = \sum_{i,j} Z_{ij} \, \chi_i(\tau) \, \chi_j(\tau)^* \,, \tag{8}$$

with non-negative integers Z_{ij}, while the Klein bottle amplitude is

$$K(t) = \sum_i K_i \, \chi_i(2\mathrm{i}t) \,. \tag{9}$$

Again there is a twisted sector – we must include open strings. For these, the partition function

$$A(t) = \sum_{a,b} n_a \, n_b \, A_{ab,i} \, \chi_i(\tfrac{\mathrm{i}t}{2}) \tag{10}$$

‡ Notice, though, that these dualities are non-perturbative as far as perturbation theory in the sigma model is concerned.

on the annulus gets projected by the contribution from the Möbius strip, i.e.

$$M(t) = \sum_a n_a \, M_{a,i} \, \chi_i(\tfrac{\mathrm{i}t+1}{2}) \,, \tag{11}$$

leading to the partition function $\frac{1}{2}(A+M)$ for unoriented open strings. (The index a labels boundary conditions for open strings and the non-negative integers n_a are so-called Chan-Paton multiplicities. They give rise to gauge symmetries in theories of open strings [24].)

There are in fact many other motivations to study boundary conditions in conformal field theories. They arise e.g. in the description of defects in quasi one-dimensional condensed matter systems, in percolation problems, and in the analysis of string propagation in the background of certain solitonic solutions, so-called D-banes. D-branes are distinguished submanifolds occurring in these solutions, similar to the position of a black hole in the Schwarzschild solution. Open strings are restricted to start and end on these submanifolds [26].

String scattering amplitudes are obtained as a perturbation series, with each term an integral over an associated CFT correlation function. In the presence of boundaries, the latter can be analyzed with methods based on the orientifold projection, which we will not describe here (for a recent review see [1]). Another approach, to be reviewed below, is based on representation theoretic structures.

5. String amplitudes and Frobenius algebras

It is an old insight that a group $\S$ G contains as much information as its category $\mathcal{C}(G)$ of finite-dimensional complex representations. (Much of the usefulness of groups in physics can be traced to this equivalence.) The category $\mathcal{C}(G)$ has the following crucial properties:
■ It has a tensor product $\otimes\colon \mathcal{C}(G) \times \mathcal{C}(G) \to \mathcal{C}(G)$, the tensor product of representations, with the trivial representation as its tensor unit.
■ It has a braiding, i.e. a family of isomorphisms $c_{V,W} \in \mathrm{Hom}(V \otimes W, W \otimes V)$ satisfying various axioms like functoriality and tensoriality; the braiding $c_{V,W}$ is simply the permutation of the two factors in the tensor product.
■ The notion of a dual, or contragredient, representation endows $\mathcal{C}(G)$ with a duality.
There are also two more technical properties:
■ There exists an object V such that every object W of $\mathcal{C}(G)$ appears as the subobject of $V^{\otimes N}$ for sufficiently large N.
■ The forgetful functor $\omega\colon \mathcal{C}(G) \to Vect_{\mathrm{fin}}(\mathbb{C})$, assigning to a representation the underlying vector space, is a fiber functor. This means in particular that it respects the tensor structure. It is the central insight of Tannaka theory (see e.g. [21] for a review) that G can be reconstructed from the abstract category $\mathcal{C}(G)$ as the group of automorphisms of the functor ω.

The approach to conformal field theory that we describe in the sequel is set up in a similar spirit – given the chiral algebra, formalized e.g. as a vertex algebra, we can determine its category $\mathcal{C}$ of representations and the associated conformal blocks. Rather than directly with the chiral algebra, we then work with the representation category $\mathcal{C}$; its objects are representations, while its morphisms are built from the conformal blocks.

For a rational CFT, the category $\mathcal{C}$ is a *modular tensor category*. (For an early review of this relationship see [25].) $\mathcal{C}$ should be thought of as a basis-independent version of the chiral data like braiding and fusing matrices and the (fractional part of) conformal weights. A modular tensor category is an abelian, semi-simple, braided tensor category with a duality, obeying also a few further conditions: it must be $\mathbb{C}$-linear; the tensor unit $\mathbf{1}$ must

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$\S$ We omit here technically important qualifiers which, depending on the chosen framework, could be "locally compact" or "algebraic".

be (absolutely) simple, i.e. $\mathrm{End}(\mathbf{1}) = \mathbb{C}\,\mathrm{id}_{\mathbf{1}}$; there must be only finitely many (isomorphism classes of) simple objects; and finally, the quadratic matrix

$$s_{ij} := \mathrm{tr}\, c_{X_i,X_j}\, c_{X_j,X_i} \tag{12}$$

must be invertible. According to the Verlinde conjecture, this matrix s also appears in the modular transformation of the characters of the vertex algebra. Modular tensor categories also arise as (truncations of) categories of representations of quantum groups and as categories of representations of weak Hopf algebras.

As an important consequence of the axioms, each modular tensor category allows for the construction of a topological field theory in three dimensions. The latter assigns to every closed oriented two-manifold $\hat{\Sigma}$ a finite-dimensional vector space $\mathcal{H}(\hat{\Sigma})$, the space of conformal blocks, on which the mapping class group $Map(\hat{\Sigma})$ acts projectively. The conformal blocks should be thought of as multivalued functions; accordingly, they are the *pre*-correlators of the local conformal field theory. Moreover, to every three-manifold M with boundary $\hat{\Sigma}$ that contains a Wilson graph, a TFT assigns a vector in $\mathcal{H}(\hat{\Sigma})$. The axioms of a TFT formalize the well-known relation between a TFT on a three-dimensional manifold and a *chiral* CFT on its boundary, a structure that is e.g. central to the description of universality classes of quantum Hall fluids (for a review, see e.g. [11]).

Our goal is now to describe CFT correlators on a surface Σ that possibly has a boundary. A major input in our construction are tools from topological field theory. To be able to apply them, we need a two-manifold $\hat{\Sigma}$ without boundaries. A natural candidate for $\hat{\Sigma}$ is the complex double of Σ. It comes with an orientation reversing involution σ such that Σ is identified with the quotient of $\hat{\Sigma}$ by the action of σ. This orientifold map is rather different from the symmetries arising in the description of twist fields of orbifolds, which are holomorphic. Indeed, it is natural [31] to view the world sheet Σ as a real scheme and $\hat{\Sigma}$ as its complexification; the orientifold map is then just the action of the Galois group $Gal(\mathbb{C}/\mathbb{R}) \cong \mathbb{Z}_2$.

With the help of $\hat{\Sigma}$ we can in particular give a concise version of the principle of *holomorphic factorization*: The correlators of the CFT on Σ are specific vectors in $\mathcal{H}(\hat{\Sigma})$, which are subject to two types of constraints:

(1) They must be invariant under the action of $Map(\Sigma) \cong Map(\hat{\Sigma})^\sigma$.

(2) They must satisfy factorization rules.

The second input in our construction is algebraic – a symmetric special Frobenius algebra in the modular tensor category $\mathcal{C}$. An algebra in $\mathcal{C}$ is an object A of $\mathcal{C}$ together with morphisms $m \in \mathrm{Hom}(A \otimes A, A)$ and $\eta \in \mathrm{Hom}(\mathbf{1}, A)$ that turn it into an algebra, i.e. m is an (associative) multiplication and η is a unit for m. A Frobenius algebra has in addition the structure of a co-algebra, i.e. there are a (co-associative) co-product $\Delta \in \mathrm{Hom}(A, A \otimes A)$ and a co-unit $\epsilon \in \mathrm{Hom}(A, \mathbf{1})$, and product and co-product are connected by the condition that

$$(m \otimes \mathrm{id}_A) \circ (\mathrm{id}_A \otimes \Delta) = \Delta \circ m = (\mathrm{id}_A \otimes m) \circ (\Delta \otimes \mathrm{id}_A). \tag{13}$$

The additional requirements that the algebra must also be symmetric and special are of a somewhat more technical nature.

In (unitary) conformal field theories, such a Frobenius algebra object is supplied by the algebra of open string states for any single given boundary condition [13]. The product comes from the operator product of boundary fields, and associativity of the OPE implies associativity of the algebra A. The Frobenius structure follows from the non-degeneracy of two-point functions of boundary fields on the disk. It is worthwhile to remark that this algebra is not necessarily (braided-) commutative.

Next we combine non-commutative algebra and 3D TFT. According to the principle of holomorphic factorization we need to select an element of $\mathcal{H}(\hat{\Sigma})$; by the principles of TFT, such a vector is determined by a three-manifold M_Σ whose boundary is the double $\hat{\Sigma}$ and a ribbon graph in M_Σ. For M_Σ we take the so-called connecting manifold [9], defined as the quotient of the interval bundle $\hat{\Sigma} \times [-1,1]$ by the $\mathbb{Z}_2$ that acts as σ on $\hat{\Sigma}$ and as $t \mapsto -t$ on the interval. The points with $t = 0$ provide a distinguished embedding of Σ into M_Σ; in fact, Σ is a retract of M_Σ, or in more intuitive terms, M_Σ is just a fattening of the world sheet Σ. (To give one example, the double of a disk is a sphere, and the orientifold map is the reflection at the equatorial plane; the connecting manifold is in this case a full 3-ball.) Concerning the prescription for the ribbon graph in M_Σ we refer to [12]. Here we merely recall that it involves a (dual) triangulation of Σ with ribbons labelled by the Frobenius algebra A.

In this framework, one can prove the consistency requirements of modular invariance and factorization. We also recover the combinatorial data (partition functions, NIM-reps [27], classifying algebras [14], . . .) that have arisen as necessary conditions in earlier work. Moreover, Morita equivalence, combined with orbifold technology, allows for an elegant proof of T-dualities for arbitrary topologies of the world sheet.

The following table, associating algebraic structures to physical concepts, can serve as a succinct summary of our results [13]:

Physical concept	Algebraic structure
Boundary condition	A-left module
Boundary field Ψ_i^{MN}	$\mathrm{Hom}_A(M \otimes i, N)$
Defect line	A-bimodule
Bulk field Φ_{ij}	$\mathrm{Hom}_{A,A}((A \otimes i)^-, (A \otimes j)^+)$
Disorder field $\Phi_{ij}^{B_1 B_2}$	$\mathrm{Hom}_{A,A}((B_1 \otimes i)^-, (B_2 \otimes j)^+)$

Our algebraization of physical concepts leads to rigorous proofs. It also allows for powerful algorithms. In particular, for constructing a full local CFT only a single non-linear constraint needs to be solved: the one encoding associativity of the Frobenius algebra A. Moreover, old physical questions amount to standard problems in algebra and representation theory:
■ The classification of CFTs with given chiral data $\mathcal{C}$ amounts to classifying Morita classes of symmetric special Frobenius algebras in the category $\mathcal{C}$. In particular, modular invariants of automorphism type are classified by the Brauer group of $\mathcal{C}$.
■ The classification of boundary conditions and defects is reduced to the standard representation theoretic problem of classifying modules and bi-modules. As a consequence, powerful methods like induced modules and reciprocity theorems are at our disposal.
■ The problem of deforming CFTs is related to the problem of deforming algebras, which is a cohomological question. For the moment, the only known results in this direction are rigidity theorems [8]: a rational CFT cannot be deformed within the class of rational CFTs.

Let us finally collect a few aspects of string theory for which symmetries play a rôle that for lack of space were omitted in the present contribution:
■ The derivation of the standard model and its symmetries from string theory. Using compactifications of the type (5), it is rather difficult to come close to the standard model. More recently, so-called brane-world models have attracted much attention (see e.g. [32]).
■ Ideas about an underlying unifying symmetry of string theory. Recently, speculations that the underlying symmetry can be described in terms of a real form of the hyperbolic Kac-Moody algebra E_{10} have been upgraded [29] to E_{11}, a Kac-Moody algebra of rank 11.
■ The construction of cosmological backgrounds using non-compact Lie groups.
■ The description of the chiral symmetry of conformal field theories. The structure of a

vertex algebra originally arose as one mathematical formalization of these symmetries. It has by now evolved in a useful mathematical structure of independent interest, with a rich structural theory. (For a recent review see [10].)

[1] C. Angelantonj and A. Sagnotti, *Open strings*, Phys. Rep. (to appear)
[2] P. Bantay, *Characters and modular properties of permutation orbifolds*, Phys. Lett. B 419 (1998) 175
[3] R.E. Behrend, P.A. Pearce, V.B. Petkova, and J.-B. Zuber, *Boundary conditions in rational conformal field theories*, Nucl. Phys. B 579 (2000) 707
[4] L. Birke, J. Fuchs, and C. Schweigert, *Symmetry breaking boundary conditions and WZW orbifolds*, Adv. Theor. Math. Phys. 3 (1999) 671
[5] R. Dijkgraaf, C. Vafa, E. Verlinde, and H. Verlinde, *The operator algebra of orbifold models*, Commun. Math. Phys. 123 (1989) 485
[6] L.J. Dixon, J.A. Harvey, C. Vafa, and E. Witten, *Strings on orbifolds*, Nucl. Phys. B 261 (1985) 678
[7] C. Dong, H. Li, and G. Mason, *Twisted representations of vertex operator algebras*, Math. Annal. 310 (1998) 571
[8] P. Etingof, D. Nikshych, and V. Ostrik, *On fusion categories*, preprint math.QA/0203060
[9] G. Felder, J. Fröhlich, J. Fuchs, and C. Schweigert, *Conformal boundary conditions and three-dimensional topological field theory*, Phys. Rev. Lett. 84 (2000) 1659; *Correlation functions and boundary conditions in RCFT and three-dimensional topology*, Compos. Math. 131 (2002) 189
[10] E. Frenkel, *Vertex algebras and algebraic curves*, Séminaire Bourbaki No. 875 (1999-2000)
[11] J. Fröhlich, B. Pedrini, C. Schweigert, and J. Walcher, *Universality in quantum Hall systems: coset construction of incompressible states*, J. Stat. Phys. 103 (2001) 527
[12] J. Fuchs, I. Runkel, and C. Schweigert, *Conformal correlation functions, Frobenius algebras and triangulations*, Nucl. Phys. B 624 (2002) 452
[13] J. Fuchs, I. Runkel, and C. Schweigert, *TFT construction of RCFT correlators I: Partition functions*, preprint hep-th/0204148, to appear in Nucl. Phys. B
[14] J. Fuchs and C. Schweigert, *A classifying algebra for boundary conditions*, Phys. Lett. B 414 (1997) 251
[15] J. Fuchs and C. Schweigert, *Lie algebra automorphisms in conformal field theory*, Contemp. Math. (2002)
[16] J. Fuchs, C. Schweigert, and J. Walcher, *Projections in string theory and boundary states for Gepner models*, Nucl. Phys. B 588 (2000) 110
[17] B.R. Greene and M.R. Plesser, *Introduction to mirror manifolds*, in: *Mirror Symmetry I*, S.-T. Yau, ed. (American Mathematical Society, Providence 1998), p. 1
[18] S. Hamidi and C. Vafa, *Interactions on orbifolds*, Nucl. Phys. B 279 (1987) 465
[19] R. Howe, *Perspectives on invariant theory: Schur duality, multiplicity-free actions and beyond*, in: *The Schur Lectures*, I. Piatetski-Shapiro and S. Gelbart, eds. (Isr. Math. Conf. Proc., Ramat-Gan 1995), p. 1
[20] C.M. Hull and P.K. Townsend, *Unity of superstring dualities*, Nucl. Phys. B 436 (1995) 507
[21] A. Joyal and R. Street, *An introduction to Tannaka duality and quantum groups*, Springer Lecture Notes in Mathematics 1488 (1991) 413
[22] V.G. Kac and I.T. Todorov, *Affine orbifolds and rational conformal field theory extensions of $W_{1+\infty}$*, Commun. Math. Phys. 190 (1997) 57
[23] Y. Kazama and H. Suzuki, *Characterization of $N=2$ superconformal models generated by the coset space method*, Phys. Lett. B 216 (1989) 112
[24] N. Marcus and A. Sagnotti, *Group theory from "quarks" at the ends of strings*, Phys. Lett. B 188 (1987) 58
[25] G. Moore and N. Seiberg, *Lectures on RCFT*, in: *Physics, Geometry, and Topology*, H.C. Lee, ed. (Plenum Press, New York 1990), p. 263
[26] J. Polchinski, *Dirichlet-branes and Ramond-Ramond charges*, Phys. Rev. Lett. 75 (1995) 4724
[27] G. Pradisi, A. Sagnotti, and Ya.S. Stanev, *Completeness conditions for boundary operators in 2D conformal field theory*, Phys. Lett. B 381 (1996) 97; see also [3]
[28] A.N. Schellekens and N.P. Warner, *Anomalies and modular invariance in string theory*, Phys. Lett. B 177 (1986) 317
[29] I. Schnakenburg and P. West, *Kac-Moody symmetries of IIB supergravity*, Phys. Lett. B 517 (2001) 421
[30] C. Schweigert, *On the classification of $N=2$ superconformal coset theories*, Commun. Math. Phys. 149 (1992) 425
[31] C. Schweigert and J. Fuchs, *The world sheet revisited*, preprint hep-th/0105266, to appear in Fields Institute Commun.
[32] A. Uranga, *From quiver diagrams to particle physics*, in: *European Congress of Mathematics*, C. Casacuberta et al., eds. (Birkhäuser, Progress in Mathematics Vol. 202, Basel 2001), p. 499