Exercise sheet # 01
Topics in representation theory WS 2017

(Ingo Runkel)

All vector spaces on this sheet are assumed to be over the complex numbers.

Exercise 1

Let $V \neq \{0\}$ be a vector space.

1. Show that V is an irreducible $GL(V)$-representation.
2. Write $\mathrm{Sym}^2(V) = \mathrm{span}\{x \otimes y + y \otimes x | x, y \in V\}$ and $\mathrm{Alt}^2(V) = \mathrm{span}\{x \otimes y - y \otimes x | x, y \in V\}$. Show that these are invariant subspaces under the $GL(V)$-action on $V \otimes V$ and that $V \otimes V = \mathrm{Sym}^2(V) \oplus \mathrm{Alt}^2(V)$.

Exercise 2

Give a precise formulation of "There is a 1-1 correspondence between representations of a group and of its group algebra" and prove it. What about intertwiners?

Exercise 3

1. Give (with proof) the implications between "irreducible" and "indecomposable".
2. Can a finite group have an infinite dimensional irreducible representation? An infinite dimensional indecomposable representation?

Exercise 4

Let λ be the Young diagram corresponding to the partition $(2, 1)$.

1. Give the element c_λ in the group algebra of S_3.
2. Give a basis of V_λ.