

## Bibliography

## Glossary of Notations



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## English translations and comments on literary citations

Cover. *The time is / what you are / and you are / what the time. / Yet that you still less / than what the time is / are.* Translation by the author.

Preface. *Hence I will be called to account for all the books I have written?* Translation by the author. Hesse's book appeared in English also under the title *Steppenwolf*.

Prologue. *To be candid, I myself, for example, have never in my life said a word to my pupils about the "meaning" of music; if there is one, it does not need my explanation. On the other I have always made a great point of having my pupils count their eighths and sixteenths nicely. Whatever you become, teacher, scholar or musician, have respect for the "meaning", but do not imagine that it can be taught.* In: *The Glass Bead Game*, Translated from the German by Richard and Clara Winston. Vintage books, London 1970.

*To write a book solely if one has to say something round is human arrogance. Aren't there, after all, more shapes [figures] than the round that all are beautiful, too? I regard the serpentine as the most suitable for a book. And I had already written in this line before I knew that Hogarth had written something about it, or before Tristram Shandy proclaimed his Manier en Ziczac or Ziczac à double Ziczac.*

Chapter 1. Marion Bradley Zimmer, *The Fall of Atlantis*. See also same author, *The Mists of Avalon*.

Chapter 2. *It is of advantage when we learn to wonder at the right objects. Often we do not wonder at the most amazing thing since it is known to us for a long time and hence seems to be self-evident.* Translation by the author. The book of von Weizsäcker also appeared in English translation under the title *The Structure of Physics*.

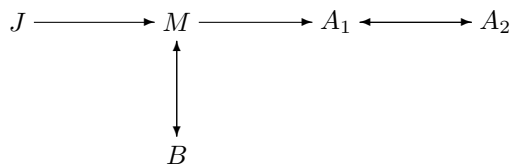
Chapter 3. Here is a somewhat longer excerpt in English: *That sort of general survey, in fact, was the special line and learning of these shiny-pated priests. They were, by their own account, very strong at generalization and at equating regional and local protecting deity with Atum-Re-Horakhte of On — a complex himself and representing a constellation of originally single numina. To make out of many one: that was their preferred activity, yes, according to them there were at bottom only two great gods: one the living, that was Hor in the Mount of Light, Atum-Re; and one on the dead, Osiris, the Eye Enthroned. But the eye also was Atum-Re, it was the disk of the sun; and so to the penetrating mind Usir was lord of the nightly bark into which, as everybody knew, Re mounted after his setting, to travel from west to east and to light the underworld. In other words, even these two great gods were at bottom one and the same. But if the shrewdness of such a general survey was admirable, not less so was the art these teachers displayed of avoiding offence; for in the midst of their assimilating activities they took care to leave intact the actual multiplicity of the Gods of Egypt. This they achieved by means of their science of the triangle [...], the beautiful figure of the conspectus. The priests of Atum plumed themselves no little on it. They had, they said, made a school with it; conspectus and comparison were going on everywhere. But only in a clumsy and uninstructed way, not in the right spirit — without intelligence, crudely and by force. Amun, for instance, the "Rich in Bulls", at Thebes in Upper Egypt, had had himself made equal to Re by his prophets and would now be called Amun-Re in his shrine. All very well, but not in the sense of the triangle and reconciliation; rather in the sense that Amun had conquered Re, had consumed him and lived in him — as though Re, so to speak, had had to name him his name! That was a brutal misuse of the doctrine, a narrow-minded effrontery quite contrary to the meaning of the triangle. Atum-Re for his part was not called the "Horizon-Dweller" for nothing; his horizon was wide and all-embracing, and all-embracing was the triangular field of his conspectus.* In: *Joseph in Egypt*. Translated from the German for the first time by H. T. Lowe-Porter. Alfred A. Knopf, New York 1938.

Chapter 4. *What is more solemn than two lines in the sand, two parallels? Look at the farthest horizon, and it has nothing to do with endlessness, really; look at the blank sea — oh, how broad it is! Or look up to the Milky Way, and it is space, in which the mind evaporates, but still it is not infinity, not really. Only they can point toward the Infinite; two lines drawn in the sand, which speak the language of Intellect.* Translation by James L. Rosenberg, *Don Juan or The Love to Geometry*, in: *Max Frisch. Three Plays*, A Mermaid Dramabook, Hill and Wang, New York 1967.

Chapter 5. The complete text is the following (Heinrich Heine, *Buch der Lieder*):

*Ein Jüngling liebt ein Mädchen,  
Die hat einen andern erwählt;  
Der andre liebt eine andre,  
Und hat sich mit dieser vermählt.  
Das Mädchen heiratet aus Ärger  
Den ersten besten Mann,  
Der ihr in den Weg gelaufen;  
Der Jüngling ist übel dran.  
Es ist eine alte Geschichte,  
Doch bleibt sie immer neu;  
Und wem sie just passiert,  
Dem bricht das Herz entzwei.*

For the non romantic, no German reading mathematician, there is a short form of this poem compressed into a diagram (“ $A \rightarrow B$ ” means “A loves B”):



Chapter 6. In the English translation by C. K. Scott Moncrieff this phrase reads as follows:

*... but often the fairest impression that remains in our minds of a favourite air is one which has arisen out of a jumble of wrong notes struck by unskilful fingers upon a tuneless piano.*

In German, the text can be translated as follows:

*... aber die schönste Ansicht, die wir von einem Werke haben, ist oft die, welche sich aus den falschen Tönen aufbaut, die ungeschickte Finger einem verstimmten Klavier entlocken.*

Chapter 7. *But Waldzell breeds the skillful Glass Bead Game players.* In loc. cit. (Prologue).

Translation of the poem *Stufen* (Stages) also by Richard and Clara Winston: *In all beginnings dwells a magic force / for guarding us and helping us to live.* Here is the complete German text of this wonderful poem:

*Wie jede Blüte welkt und jede Jugend  
Dem Alter weicht, blüht jede Lebensstufe,  
Blüht jede Weisheit auch und jede Tugend  
Zu ihrer Zeit und darf nicht ewig dauern.  
Es muß das Herz bei jedem Lebensrufe  
Bereit zum Abschied sein und Neubeginne,  
Um sich in Tapferkeit und ohne Trauern  
In andre, neue Bindungen zu geben.  
Und jedem Anfang wohnt ein Zauber inne,  
Der uns beschützt und der uns hilft, zu leben.*

*Wir sollen heiter Raum um Raum durchschreiten,  
An keinem wie an einer Heimat hängen,  
Der Weltgeist will nicht fesseln uns und engen,  
Er will uns Stuf' um Stufe heben, weiten.  
Kaum sind wir heimisch einem Lebenskreise  
Und traulich eingewohnt, so droht Erschlaffen,  
Nur wer bereit zu Aufbruch ist und Reise,  
Mag lähmender Gewöhnung sich entrafen.*

*Es wird vielleicht auch noch die Todesstunde  
Uns neuen Räumen jung entgegen senden,  
Des Lebens Ruf an uns wird niemals enden ...  
Wohlan denn, Herz, nimm Abschied und gesunde!*

Chapter 8. This is a remark of Patrick Süskind on his writing of the play *The Double Bass: I could fall back upon my own experience in so far as I spend the greater part of my life in ever decreasing rooms which to leave I find harder and harder. Still I hope to find some day a room that is so small and encloses me so tightly that I can carry it with me when leaving.* (Translation by the author).

Chapter 9. *Thus his path had been a circle, or an ellipse or spiral or whatever, but certainly not straight; straight lines evidently belonged only to geometry, not to nature and life.* In: *The Glass Bead Game*, loc. cit.

Chapter 10. “But that is true?” says Lisa. “The princess wanted a piece of cotton, as large as a pillow? And when she got it, she recovered?” I see that Jacob’s mouth becomes broader, he says: “Not quite. She wished a cloud. The funny part is that she thought clouds are made of cotton, and that’s why she was content with the cotton”. Lina looks out for a while, amazed as it seems to me, before she asks him: “But clouds are not made out of cotton?”. Publication in English as Jurek Becker: *Jacob the Liar*.

Chapter 11. Eduard Möricke’s *Elfenlied* has been set to music by Hugo Wolf (see also the music citations). In the German–English edition by C. F. Peters Corporation, the translation reads (partly) as follows: “Elev’n o’ clock” the watchman cries “hear me!” Asleep in the woods quite a wee elf lies - in fright upstarts he. He knew not whence that loud sound came, he thought the nightingale called his name, or that Silpelit might have required him. [...] Poor elf, say, will that now do? Cuckoo!

Chapter 12. *He knew that his memory of the piano falsified still further the perspective in which he saw the music, that the field open to the musician is not a miserable stave of seven notes, but an immeasurable keyboard (still, almost all of it, unknown), on which, here and there only, separated by the gross of its unexplored tracts, some few among the millions of keys, keys of tenderness, of passion, of courage, of serenity, which compose it, each one differing from all the rest as one universe differs from another, have been discovered by certain great artists who do us the service, when they awaken in us the emotion corresponding to the which they have found, of shewing us what richness, what variety lies hidden, unknown to us, in that great black impenetrable night, discouraging exploration, of our soul, which we have been content to regard as valueless and and void.*

Chapter 13. Robert Musil, *The man without qualities*. Translation by [...]: *A barometric low hung over the Atlantic. It moved eastward toward a high-pressure area over Russia without as yet showing any inclination to bypass this high in a northerly direction. The isotheres and isotherms were functioning as they should. The air temperature was appropriate relative to the annual mean temperature and to the aperiodic monthly fluctuations of the temperature. The rising and setting of the sun, the moon, of Venus, the rings of Saturn, and many other significant phenomena were all in accordance with the forecasts in the astronomical yearbooks. The water vapour in the air was at its maximal state of tension, while the humidity was minimal. In a word that characterises the facts accurately, even if a bit old fashioned: It was a fine day in August 1913.*

Chapter 14. *That we have been on the moon [...] we can only imagine if we do not look up to it.* There is also an English version published under the title *The Mathematics of Nina Gluckstein*.

Chapter 15. *Thereafter more and more new relations, analogies and correspondences were discovered among the abstract formulas obtained in this way. Each discipline which seized upon the Game created its own language of formulas, abbreviations and possible combinations. Everywhere, the elite intellectual youth developed a passion for these Games, with their dialogues and progression of formulas.* In: *The Glass Bead Game*, loc. cit.

Chapter 16. Maurice Maeterlinck’s sentences are taken from the fourth of his collection of 13 essays called *La Morale Mystique*. An English translation by Alfred Sutro appeared in 1897: *How strangely do we diminish a thing as soon as we try to express it in words! We believe we have dived down to the most unfathomable depths, and when we reappear on the surface, the drop of water that glistens on our*

*trembling finger - tips no longer resembles the sea from which it came. We believe we have discovered a grotto that is stored with bewildering treasure; we come back to the light of day, and the gems we have brought are false – mere pieces of glass – and yet does the treasure shine on, unceasingly, in the darkness!.*

These words are much better known to German reading persons as the *motto* of Robert Musil's novel *Verwirrungen des Zöglings Törleß*. In his wonderful translation they sound as follows: *Sobald wir etwas aussprechen, entwerten wir es seltsam. Wir glauben in die Tiefe der Abgründe hinabgetaucht zu sein, und wenn wir wieder an die Oberfläche kommen, gleicht der Wassertropfen an unseren bleichen Fingerspitzen nicht mehr dem Meer, dem es entstammt. Wir wähnen eine Schatzgrube wunderbarer Schätze entdeckt zu haben, und wenn wir wieder ans Tageslicht kommen, haben wir nur falsche Steine und Glasscherben mitgebracht; und trotzdem schimmert der Schatz im Finstern unverändert.*

Chapter 17. *Over there, beyond the incense, there where the air is clear and joyous and the revelations begin! That is no realm of whims and fancies, Roderigo, but that which is meaningful today is meaningful tomorrow, and when I have ceased to breathe, its meaning still shall live on forever – you. Only the serene spirit knows what blessedness is; everything else is but glitter and deception, believe me, worth the holding.* In loc. cit.

Chapter 18. *In nature we find not words, but only the initial letters of words, and if we then attempt to read them we find that the new so-called words are again merely the initial letters of other words.* Translation by R. J. Hollingdale.

Chapter 19. *No, said Rambert with bitterness, you can't understand. You speak the language of reason, you are in abstraction. But there where some saw abstraction, others saw truth.* Translation by the author. There are several translations into English published under the title *The plague*.

Epilogue. *Lichtenberg: It frequently happened to me that - when I let something go to print - I noticed at the very end when nothing could be modified anymore that I could have said everything much better, and even more that I had forgotten main facts. This annoyed me very much.*

*Muschg: We had been in China, still a couple of names were flying with us; soon they as well would be merely declarations. And some day China will not lie anymore so shining in our memory as it once was lying deep in our expectation.* Translations by the author.

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<sup>1</sup>It was Y. Namikawa who told me about J.–P. Rampal’s *Japanese Flute Favorites* arranged by Akio Yashiro and helped me to purchase the notes in Kyoto. In 2017, I visited with my wife Kyoko and my friends Hideaki and Yoko Kazama the home of Rentaro Taki on the island of Kyushu and played this piece on the nearby Oka castle next to the statue of Taki. The photograph on the next page was taken at this occasion.

<sup>2</sup>The so called “*Champagne Aria*”.

<sup>3</sup>The composer, pianist and musical director of the *Gewandhaus* in Leipzig was born in Altona near Hamburg (now a suburb of Hamburg) in 1824.

<sup>4</sup>The fairy tale of the same title written by the German poet of aristocratic Huguenot extraction, Friedrich de la Motte–Fouqué, was published in 1811. It was used as a libretto by not less than 22 composers from 1816 to 1917. Ernst Theodor Amadeus Hoffmann’s *Undine* of 1816 is the very first of all Romantic operas.

<sup>5</sup>The photo on my homepage was this piece with Yoko Kazama at the *Riemenhaus* in Fukuoka; c. f. <http://www.math.uni-hamburg.de/home/riemenschneider/>

<sup>6</sup>If you don’t like it since you heard it much to often: it’s very good for warming up, your flute, your breath, your diaphragm, even your soul.

<sup>7</sup>This piece has been “instrumental” for my collaboration with David Eisenbud. See [13–12] and [13–15].

<sup>8</sup>The so called “*Mirror Aria*”.

<sup>9</sup>I had the pleasure that some pianists whom I met at mathematical occasions like, among others, Wolf Hofmann, Ludger Kaup, Joseph Steenbrink and Kei-ichi Watanabe, joined me in the task to perform this piece and/or many others.

<sup>10</sup>This *Leitmotiv* reflects Undine’s desire for a human soul.



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## Epilogue

*Es ist mir öfter begegnet, daß ich, wenn ich etwas habe drucken lassen, erst ganz am Ende, wenn sich nichts mehr ändern ließ, bemerkt habe, daß ich alles hätte besser sagen können, ja, daß ich Hauptumstände vergessen hatte. Dies ärgerte mich oft sehr.*

(Georg Christoph Lichtenberg)

*Wir waren in China gewesen, noch flogen ein paar Namen mit uns; bald würden auch sie nur noch Behauptungen sein. Und eines Tages wird China nicht mehr so hell im Gedächtnis liegen, wie es einmal in der Tiefe der Erwartung gelegen hatte.*

(Adolf Muschg:

*Baiyun oder die Freundschaftsgesellschaft)*

