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A note on the toric duality between the cyclic quotient surface singularities $A_{n,q}$ and $A_{n,n-q}$. (English summary)

Singularities in geometry and topology, 161–179, *IRMA Lect. Math. Theor. Phys.*, 20, *Eur. Math. Soc.*, *Zürich*, 2012.

Let n and q be coprime integers, $1 \le q < n$, ζ_n a primitive complex root of unity, and $\Gamma_{n,q}$ the cyclic group generated by the diagonal matrix $\mathrm{diag}(\zeta_n, \zeta_n^q)$. Denote by $A_{n,q}$ the quotient surface singularity $\mathbb{C}^2/\Gamma_{n,q}$. Then, the singularity is an affine toric variety and, as the author points out in Section 3 of the paper under consideration, the singularities $A_{n,q}$ and $A_{n,n-q}$ are dual to each other as affine toric varieties. This means that if $A_{n,q}$ is determined by a lattice $M \simeq \mathbb{Z}^2$ and a rational polyhedral cone σ in $M \otimes \mathbb{R}$, then $A_{n,n-q}$ is determined by the dual lattice $N = \mathrm{Hom}(M, \mathbb{Z})$ and the dual cone $\sigma^\vee \subset N \otimes \mathbb{R}$.

Quotient singularities of the form $A_{n,q}$ are well studied in the literature [see, e.g., J. Stevens, in *Deformations of surface singularities*, 163–201, Bolyai Soc. Math. Stud., 23, Springer, Berlin, 2013; and references therein]. It is known that the versal deformation $\mathfrak{X}_{n,q}^{\mathrm{vers}}$ of $A_{n,q}$ has in general a reducible base space. There exists always one component of $\mathfrak{X}_{n,q}^{\mathrm{vers}}$ which is the versal deformation space for deformations of $A_{n,q}$ which possess a simultaneous resolution after finite base change. It is called the *Artin deformation*. There is also the so-called *monodromy covering* $\mathfrak{Y}_{n,q}$ of the Artin deformation which is versal for deformations of $A_{n,q}$ which can be resolved simultaneously without base change. J. A. Christophersen [in *Singularity theory and its applications, Part I (Coventry, 1988/1989)*, 81–92, Lecture Notes in Math., 1462, Springer, Berlin, 1991; MR1129026] showed that the total space $\mathfrak{Y}_{n,q}$ is an affine toric variety, and M. Hamm in his dissertation [*Die verselle Deformation zyklischer Quotientensingularitäten: Gleichungen und torische Struktur*, Univ. Hamburg, 2008] proved that $\mathfrak{Y}_{n,q}$ and $\mathfrak{Y}_{n,n-q}$ are also dual to each other as affine toric varieties. In the present paper the author gives a proof of Hamm's result in the special cases $q = n-1$ (then $A_{n,q}$ is isomorphic to the singularity A_{n-1}) and $q = 1$ (then $A_{n,q}$ is isomorphic to a cone over the twisted rational curve). The aim of this new proof is to make the toric duality between $\mathfrak{Y}_{n,q}$ and $\mathfrak{Y}_{n,n-q}$ more geometrically visible.

{For the collection containing this paper see MR3077315}

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