

MR2087059 (2006b:32041) 32S40
Riemenschneider, Oswald (D-HAMB-SM)

The monodromy covering of the versal deformation of cyclic quotient surface singularities. (English summary)

Complex analysis in several variables—Memorial Conference of Kiyoshi Oka's Centennial Birthday, 275–282, Adv. Stud. Pure Math., 42, Math. Soc. Japan, Tokyo, 2004.

From the introduction: “By studying the special case of cyclic quotient surface singularities several general aspects of deformation theory of complex-analytic singularities have been detected, e.g. the existence of many components of the base space of the versal deformation (which we also call the versal base space for short) and their monodromy coverings and the existence of embedded components. Moreover, the (necessarily) smooth reduced components, the deformations thereon including the discriminant and the adjacencies and the monodromy coverings, can be explicitly described and are very well understood.

“The versal base space itself has—in the first interesting case of embedding dimension $e = 5$ —quite simple equations. Later, J. Arndt [“Verselle Deformationen zyklischer Quotientensingularitäten”, dissertation, Univ. Hamburg, Hamburg, 1988; Zbl 0643.32007] calculated those equations for embedding dimension 6 and gave a ‘quasi-algorithmic’ structure theorem for the general case. In his dissertation, S. Brohme [“Monodromieüberlagerung der versellen Deformation zyklischer Quotientensingularitäten”, dissertation, Univ. Hamburg, Hamburg, 2002; per bibl.] proposed an explicit algorithm to produce equations in the cyclic case which are more closely related to the continued fractions than those given in [T. de Jong and D. van Straten, *J. Algebraic Geom.* **3** (1994), no. 1, 117–172; [MR1242008](#)] and proved that his algorithm really leads to correct equations up to embedding dimension 8. It should also be mentioned that K. Miyajima [“Deformation of CR structures on a link of normal isolated singularity”, manuscript, per bibl.] has done some calculations on the versal deformation space by means of the deformation theory of CR-structures.

“However, all these sets of equations are extremely complicated (therefore, they are not reproduced here due to lack of space). In particular, it is almost impossible to draw any geometric conclusions from them. Despite the beautiful ‘picture method’ of de Jong and van Straten, there was in my opinion a ‘satisfactory’ construction of the versal deformation—in the case of cyclic quotients—in terms of combinatorics, i.e. in terms of the continued fraction associated to such a singularity, still missing. In order to remedy this unpleasant situation, I sketched in August 1996 an explicit construction of (a finite covering of) the reduced versal deformation space (the main idea is already contained in [O. Riemenschneider, *Sūrikaiseikikenkyūsho Kōkyūroku* No. 807 (1992), 93–118; [MR1254047](#)]). In the following I shall state the result after some preparatory notions and remarks; a proof is contained in [S. Brohme, *op. cit.*]. Due to explicit computer algebra calculations via Singular in small embedding dimensions with the help of Brohme’s equations, I am convinced that also the embedded components can successfully be ‘attached’ to this construction.”

{For the collection containing this paper see [MR2087033](#)}