MR0437808 (55 #10730) 32G10 32C40
Riemenschneider, Oswald
Familien komplexer Räume mit streng pseudokonvexer spezieller Faser.
Comment. Math. Helv. **51** (1976), *no. 4,* 547–565.

The author gives the following results, which generalize to higher dimensions some aspects of work of E. Brieskorn [Actes du Congrès International des Mathématiciens (Nice, 1970), Tome 2, pp. 279–284, Gauthier-Villars; #10720 above], M. Artin [J. Algebra **29** (1974), 330–348; MR0354665] and of the author [Manuscripta Math. **14** (1974), 91–99; MR0414930]. Let $\tilde{\pi}\colon \tilde{Z}\to S$ be a holomorphic map between complex spaces. Let $0\in S$. Let $\tilde{Z}_0=\tilde{\pi}^{-1}(0)$ be strictly pseudoconvex. Let K be compact in $\tilde{Z}_0$. Then there are open sets $U\subset\tilde{Z}$ and $V\subset S$ with $K\subset U$, $0\in V$ and $\tilde{\pi}(U)\subset V$ such that $\tilde{\pi}|U\colon U\to V$ is a 1-convex map. So one can assume that $\tilde{\pi}$ is 1-convex. Let $\sigma\colon\tilde{Z}\to Z$ be the Remmert reduction of Z with $\tilde{\pi}=\pi\circ\sigma$. Let $\mathcal{F}$ be a coherent sheaf on $\tilde{Z}$ which is flat over S near $\tilde{Z}_0$. If $H^1(\tilde{Z}_0,\mathcal{F}_0)=0$ or if $\dim H^1(\tilde{Z}_s,\mathcal{F}_s)$ is constant in a neighborhood of 0 and S is reduced near 0, then $\sigma_*\mathcal{F}$ is π-flat near the fiber $Z_0=\pi^{-1}(0)$. Typically $\mathcal{F}$ is $\mathcal{O}$, the structure sheaf. Suppose that $H^2(\tilde{Z}_s,\mathcal{O}_s)$ has constant dimension. Then there is a maximal reduced subspace S_a of S such that restriction to S_a makes π_α a flat deformation of Z_0; here

$$S_a=\{s\in S\colon \dim H^1(\tilde{Z}_s,\mathcal{O}_s)=\dim(\tilde{Z}_0,\mathcal{O}_0)\}.$$

If $H^2(\tilde{Z}_0,\mathcal{O}_0)=0$ and $\tilde{\pi}$ is a versal deformation of the germ of $\tilde{Z}_0$ near its exceptional set, then π_a is the versal deformation of Z_0 which can be simultaneously resolved (without base change) and with initial resolution $\tilde{Z}_0$.

The existence of such a versal $\tilde{\pi}$ is now known in some cases to be published by D. Leistner (“Vollständigkeitssatz für Deformationen von streng pseudokonvexen Mannigfaltigkeiten”, preprint, 1976) and by the reviewer (“Deformations of two-dimensional pseudoconvex manifolds”, to appear in Proceedings of the Conference on Complex Analysis (Cortona, 1977)). *H. B. Laufer*