MR0414930 (54 #3022) 32C40
Riemenschneider, Oswald
Bemerkungen zur Deformationstheorie nichtrationaler Singularitäten. (German. English summary)
Manuscripta Math. **14** (1974), 91–99.

Let $\pi_0\colon M_0 \to X_0$ be a resolution of a normal complex analytic space with $x_0 \in X_0$ as its only singular point. Let $\pi\colon M \to S$ be a deformation of M_0 with $M_0 = \pi^{-1}(s_0)$. The question considered in this paper is: when does M blow down to a deformation of X_0? Affirmative answers to this question provide one important way of deforming singularities. By shrinking M and S, one can assume that π is a 1-convex mapping [the author, Rice Univ. Studies **59** (1973), no. 1, 119–130; MR0355101] and that M is holomorphically convex. M then blows down to its Remmert reduction X, also yielding a map $\omega\colon X \to S$. Then $\omega^{-1}(s_0) = X_0$ if and only if every holomorphic function on $\pi^{-1}(s_0)$ defined in a neighborhood of the exceptional set can be extended holomorphically to a function on nearby points of X. In particular, $\omega^{-1}(s_0) = X_0$ occurs if S is reduced, π is flat and $\dim H^1(\pi^{-1}(s), \mathcal{O}(\pi^{-1}(s))$ is constant for all s. J. M. Wahl [Ann. of Math. (2) **104** (1976), no. 2, 325–356] has obtained similar results in the algebraic category.

Examples are also given of deformations π that cannot be blown down to deformations of X.

H. B. Laufer