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Riemenschneider, Oswald
Deformationen von Quotientensingularitäten (nach zyklischen Gruppen). (German)
Math. Ann. **209** (1974), 211–248.

This paper studies the deformations of those normal two-dimensional singularities that can be realized as the quotient of $\mathbf{C}^2$ by the action of a finite cyclic group. As shown by E. V. Brieskorn [Invent. Math. **4** (1967/68), 336–358; MR0222084], these cyclic quotient singularities can also be characterized as having resolutions whose dual graphs are unbranched lines of rational curves. These resolutions appear prominently in the work of H. W. Jung [J. Reine Angew. Math. **133** (1908), 289–314] and F. Hirzebruch [Math. Ann. **126** (1953) 1–22; MR0062842].

Given such a singularity p, the author calculates explicit generators for its defining ideals and also for its relations. This is done using two parametric representations for the local ring of p: one is the invariant functions on $\mathbf{C}^2$ under the group action and the other is global functions on the Jung-Hirzebruch resolution. Using these generators, a special deformation is given explicitly for p. After a base change, this deformation can be simultaneously resolved, in the manner of Brieskorn [Actes du Congrès International des Mathématiciens (Nice, 1970), Tome 2, pp. 279–284, Gauthier-Villars, Paris, 1971]. The singularities in this deformation are also cyclic quotient singularities. They can all be described as having resolutions whose dual graphs can be obtained from the dual graph for a resolution of p by a sequence of either (i) deleting exceptional curves or (ii) replacing two intersecting exceptional curves with self-intersection numbers $-b_1$ and $-b_2$ by an exceptional curve with self-intersection number $-b_1 - b_2 + 2$. In particular, p is smoothable. In case the embedding dimension for p is 3 or 4, then the special deformation is also the versal deformation of p. For an embedding dimension of 5, the versal deformation of p is explicitly calculated. The special deformation is seen to be a proper component of the versal deformation. An example is given in which the versal deformation contains singularities which are not in the special deformation. This contrasts with the examples of H. C. Pinkham [J. Algebra **30** (1974), 92–102; MR0347822].

The author concludes with some questions and conjectures. Two conjectures, that the special deformation is versal with respect to resolvable deformations and that an analogous statement holds for all rational singularities, have since been answered affirmatively by M. Artin [J. Algebra **29** (1974), 330–348; MR0354665]. *H. B. Laufer*