MR0316742 (47 #5290) 32B05 13H05 13H10 13J05 13J15 32-02 32A05
Grauert, H. [Grauert, Hans]; Remmert, R. [Remmert, Reinhold]
★**Analytische Stellenalgebren. (German)**
Unter Mitarbeit von O. Riemenschneider.
Die Grundlehren der mathematischen Wissenschaften, Band 176.
Springer-Verlag, Berlin-New York, 1971. ix+240 *pp.* DM 64.00; $18.50.

The title essentially refers to the local algebras of complex analytic spaces. Thus there is no special consideration of local rings (although this could have been done in some sections) and also none of topics like localization or the case of a non-archimedean ground field. In a sense, this book represents the punctual theory of analytic spaces. The local and (for Stein spaces) the global theory will be treated in a subsequent volume "Kohärente analytische Garben". Accordingly, there is no mention of analytic spaces, or their points or neighborhoods. Since germs of analytic functions and sets are also not introduced (although they belong to the punctual theory), the reader will find it difficult to interpret theorems and definitions geometrically as recommended in the introduction. Therefore the reviewer thinks that the reader should have a background in analytic or algebraic geometry. Of course a basic knowledge of commutative algebra and set-theoretic topology is required. Requirements in commutative algebra beyond this are found in the appendix.

Here is a detailed summary of the contents: Let k be a complete valued field subject to various conditions (all of which are satisfied by the complex number field). In Chapter I the convergent power series algebras $k\{X\}=k\{X_1,\cdots,X_n\}$ are considered. The t-norms ($t\in\mathbf{R}_+{}^n$):

$$\|\sum a_{\nu_1\cdots\nu_n}X_1{}^{\nu_1}\cdots X_n{}^{\nu_n}\|_t=\sum|a_{\nu_1\cdots\nu_n}|t_1{}^{\nu_1}\cdots t_n{}^{\nu_n}$$

are the basis of elegant proofs of the Weierstraßdivision and preparation theorems with their important immediate consequences (e.g., $k\{X\}$ is Noetherian, factorial and Henselian). The "sequential topology" of $k\{X\}$ and of finitely generated $k\{X\}$ modules is discussed in detail. This topology is defined by the collection of t-norms. It is described, e.g., by M. Jurchescu [Bull. Soc. Math. France **93** (1965), 129–153; MR0197774], but has hardly been used until now, on the one hand because of its shocking behaviour and, on the other hand, because it usually suffices to consider just "analytically convergent" sequences.

In Chapter II these considerations are extended to the category of analytic local algebras $A=k\{x\}/\mathfrak{a}$ and to the category of finite A-modules. Chevalley's theory of dimension of A-modules is developed with the aid of the "Active Lemma": If $f\in A$ is an active non-unit, then $\dim A/Af=\dim A-1$. Here f is said to be active if no minimal prime ideal of A contains f (every nonzero-divisor is active). Krull dimension and normalization are also discussed.

Chapter III is devoted to special concepts for analytic local algebras like homological codimension, syzygies, derivations and differential modules, and analytic tensor products. No knowledge of homological algebra is required.

The book is written very carefully and precisely. Interesting examples are described in detail. The book is recommended to everyone interested in analytic algebras whos has the prerequisites mentioned above. *K. Wolffhardt*