MR0302938 (46 #2081) 32C35 32L10
Grauert, Hans; **Riemenschneider, Oswald**
Verschwindungssätze für analytische Kohomologiegruppen auf komplexen Räumen. (German)
Invent. Math. **11** (1970), 263–292.

This paper contains the proofs for results announced previously under the same title [*Several complex variables*, I (Proc. Conf., Univ. of Maryland, College Park, Md., 1970), pp. 97–109, Springer, Berlin, 1970; MR0273066].

The vanishing theorems of K. Kodaira [Proc. Nat. Acad. Sci. U.S.A. **39** (1953), 1268–1273; MR0066693] and S. Nakano [J. Math. Soc. Japan **7** (1955), 1–12; MR0073263] are generalized as follows. Let X be a Moĭšezon space, i.e., an irreducible compact (reduced) analytic space of dimension n that has n independent meromorphic functions. Let $\pi\colon \hat{X} \to X$ be a resolution of X. Let $K(\hat{X})$ be the canonical sheaf of $\hat{X}$. Let $K = K(X) = \pi_0(K(\hat{X}))$ be defined as the canonical sheaf of X. K is shown to be independent of the choice of π. As introduced by the first author [Math. Ann. **146** (1962), 331–368; MR0137127], every coherent analytic sheaf $\mathcal{S}$ has a corresponding linear space L. For suitably small open sets U in X, L is locally a subset of $U \times \mathbf{C}^q$, for q depending on U. $\mathcal{S}$ is said to be semi-positive when L has a Hermitian metric that may be induced locally from a semi-positive, in the sense of Nakano, metric on $U \times \mathbf{C}^q$. $\mathcal{S}$ is said to be quasi-positive if it is semi-positive on some open dense subset of X. Suppose now that $\mathcal{S}$ is quasi-positive and torsion-free, Let $\pi^*\mathcal{S}$ be the pull-back of $\mathcal{S}$ to $\hat{X}$. Let $\mathcal{T}$ be the torsion subsheaf of $\pi^*\mathcal{S}$ and $\mathcal{S}\circ\pi = \pi^*\mathcal{S}/\mathcal{T}$. $\mathcal{S}\cdot K$, defined as $\pi_0(\mathcal{S}\circ\pi\otimes K(\hat{X}))$, is independent of the choice of π. The main theorem is that $H^\nu(X, \mathcal{S}\cdot K) = 0$ for $\nu \geq 1$.

The main steps of the proof are as follows. $\hat{X}$ can be chosen to be projective-algebraic [B. G. Moĭšezon, Izv. Akad. Nauk SSSR Ser. Mat. **31** (1967), 1385–1414; MR0222917]. $\mathcal{S}\circ\pi$ can be chosen to be locally free [H. Rossi, Rice Univ. Studies **54** (1968), no. 4, 63–73; MR0244517]. $\mathcal{S}\circ\pi$ is shown to be quasi-positive. Nakano's proof extends to quasi-positive vector bundles over Kähler manifolds. $\pi_\nu(\mathcal{S}\circ\pi\otimes K(\hat{X}))$ is shown to be 0 for $\nu \geq 1$ by induction on ν. Essential use is made here of the fact that $\hat{X}$ has a positive line bundle F. Tensoring with high powers of F simplifies the cohomology [the first author, op. cit.]. The theorem now follows.

The canonical sheaf used in this paper is in general different from Grothendieck's canonical sheaf $K^*(X)$ at the singular points of X. An example is given showing that the vanishing theorem of this paper need not hold with K replaced by $K^*(X)$.

Now suppose that X is a strongly pseudoconvex manifold. Let $\pi\colon X \to Y$ be the blowing down of the exceptional set E in X. Then $\pi_\nu(K(X)) = 0$ for $\nu \geq 1$. Y has only isolated singularities. By results of M. Artin [Inst. Hautes Études Sci. Publ. Math. No. 36 (1969), 23–58; MR0268188] these can be locally made algebraic. Then previous methods of this paper yield the proof. Finally, when X is Kähler and Y is a manifold, the remaining analytic $H^{p,q}$-cohomology for X is strongly related to topological data via the theorem $H^l(E;\mathbf{C}) \approx \oplus H^{p,q}(X)$, $p+q=l$ and $p,q \geq 1$. *H. B. Laufer*