**MR0212210 (35 #3084)** 32.22
**Riemenschneider, Oswald**
**Über den Flächeninhalt analytischer Mengen und die Erzeugung $k-$pseudokonvexer Gebiete. (German)**
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Results of K. Oka [Japan. J. Math. **32** (1962), 1–12; MR0159032] are extended from analytic sets of pure condimension 1 to analytic sets of pure dimension $k$. At first different but equivalent definitions of $k$-pseudo-convexity are given. For $r > 0$, define $D(r) = \{z : |z| < r\}$ and $D^n(r) = D(r) \times \cdots \times D(r)$ ($n$-factors). Let $\overline{D}^n(r)$ be the closure of $D^n(r)$. Define ${D_0}^n(r) = D^n(r) - \{0\}$. Let $G_0$ be an open subset of a pure $n$-dimensional complex manifold $G$. Then $G_0$ is said to be $k$-pseudo-convex in $G$ if the following condition holds: "If $a \in \overline{G}_0 - G_0$ and if $f: U \to V$ is a biholomorphic map of the open neighborhood $U$ of $\{0\} \times \overline{D}^{n-k}(r)$ in $\mathbf{C}^n$ onto an open subset $V$ of $G$ with $f(0) = a$ and with $f(\{0\} \times {D_0}^{n-k}(r)) \subseteq G_0$, then $r_1 > 0$ exists such that

$$f(\{z\} \times D^{n-k}(r)) \cap (G - G_0) \neq \varnothing$$

for each $z \in D^k(r_1)$."

For $0 < r_1 < r$ and $0 < {r_1}' < r'$ define

$$P = P\left(\frac{k}{n}\middle|\frac{r}{r'}\right) = D^k(r) \times D^{n-k}(r),$$

$$Q\left(\frac{k}{n}\middle|\frac{r}{{r_1}', r'}\right) = D^k(r) \times (D^{n-k}(r') - \overline{D}^{n-k}({r_1}')),$$

$$H^0\left(\frac{k}{n}\middle|\frac{r_1, r}{{r_1}', r'}\right) = Q\left(\frac{k}{n}\middle|\frac{r}{{r_1}', r'}\right) \times P\left(\frac{k}{n}\middle|\frac{r_1}{r'}\right).$$

Let $\Delta_m$ be the set of all integers $p$ with $1 \leq p \leq m$. Take $s$ with $0 < s < k$. Let $S$ be the set of all injective and increasing maps $\sigma: \Delta_s \to \Delta_k$, where $0 < s < k$. For $\sigma \in S$ and $p \in \Delta_k$ define $m_\sigma(p) = r$ if $p \in \sigma(\Delta_s)$ and $m_\sigma(p) = r_1$ if $p \in \Delta_k - \sigma(\Delta_s)$. For $\sigma \in S$ define

$$Q^\sigma\left(\frac{k}{n}\middle|\frac{r_1, r}{r'}\right) = D(m_\sigma(1)) \times \cdots \times D(m_\sigma(k)) \times P\left(\frac{k}{n}\middle|\frac{r_1}{r'}\right),$$

$$H^s = H^s\left(\frac{k}{n}\middle|\frac{r_1, r}{{r_1}', r'}\right) = Q\left(\frac{k}{n}\middle|\frac{r}{{r_1}', r'}\right) \cup \bigcup_{\sigma \in S} Q^\sigma\left(\frac{k}{n}\middle|\frac{r_1, r}{r'}\right).$$

Then $(H^s, P)$ is said to be an $(s, k, n)$-Hartogs figure. The open subset $G_0$ of $G$ is said to be $(s, k)$-pseudo-convex in $G$ if the following property is true: "If $a \in \overline{G}_0 - G_0$, if $(H^s, P)$ is an $(s, k, n)$-Hartogs figure and if $f: U \to V$ is a biholomorphic map of an open neighborhood $U$ of $\overline{P}$ onto an open subset $V$ of $G$ with $f(0) = a$ such that $f(H^s) \subseteq G_0$, then $f(P) \subseteq G_0$." Theorem: "If $0 \leq s < k \leq n$, then $G_0$ is $(s, k)$-pseudo-convex in $G$ if and only if $G_0$ is $k$-pseudo-convex in $G$." The advantage of this result is that $(k-1, k)$-pseudo-convexity pseudo-convexity is better adapted to induction.

Again, let $G$ be a pure $n$-dimensional complex manifold with a countable base of open sets. Let $\chi$ be the $(1,1)$-form associated to a Hermitian metric on $G$. Define $\chi_k = (1/k!)\chi \wedge \cdots \wedge \chi$ ($k$-times). A real $k$-divisor $A = \{A_\lambda, a_\lambda\}_{\lambda \in \Lambda}$ consists of a locally finite family $\{A_\lambda\}_{\lambda \in \Lambda}$ of irreducible analytic sets of dimension $k$ on $G$ and a family $\{c_\lambda\}_{\lambda \in \Lambda}$ of real numbers. The $k$-divisor $A$ is said to be positive if all $c_\lambda > 0$. The $k$-divisor is said to be integral if all $c_\lambda$ are integers. For an open, relative compact subset $U$ of $G$ the volume

$F_\chi(A;U) = \sum_{\lambda\in\Lambda} c_\lambda \int_{A_\lambda \cap U} \chi_k$ exists. A set $\mathfrak{A}$ of positive $k$-divisors on $G$ is said to be bounded at $c \in G$ if an open, relative compact neighborhood $U$ of $c$ and a number $K > 0$ exist such that $F_\chi(A;U) \leq K$ for all $A \in \mathfrak{A}$. This condition is independent of the choice of $\chi$. The set $G(\mathfrak{A})$ of all points $c$ of $G$ where $\mathfrak{A}$ is bounded is obviously open and is called the set of local boundness. The author's main result: "$G(\mathfrak{A})$ is $k$-pseudo-convex."

In the case $n-1=k=1$, this theorem is due to K. Oka [loc. cit.] and to T. Nishino [J. Math. Kyoto Univ. **1** (1961/62), 357–377; MR0148945]. In the case of a set $\mathfrak{A}$ of positive integral $(n-1)$-divisors, the domain $G[\mathfrak{A}]$ of normality is defined. Montel's theorem asserts $G[\mathfrak{A}] = G(\mathfrak{A})$. In this case, T. J. Barth ["The normality domain of a set of divisors", Ph.D. dissertation, Univ. Notre Dame, Notre Dame, Ind., 1966] proved independently that $G(\mathfrak{A})$ is pseudo-convex. Also, O. Fujita [J. Math. Soc. Japan **16** (1964), 379–405; MR0178158] gave a direct proof for the pseudo-convexity of $G[\mathfrak{A}]$. He considered the case of $G(\mathfrak{A})$ using induction [J. Math. Kyoto Univ. **4** (1964/65), 627–635; MR0180700]. In his first paper, Fujita introduced a certain notion of normality for a set of positive integral $k$-divisors and proved that $G[\mathfrak{A}]$ is $k$-pseudo-convex. However, a Montel theorem for $k$-divisors is not yet proved.

{The reviewer has formulated the results in a somewhat more general fashion than is done in the paper under review, where only open subsets $G$ of $\mathbf{C}^n$ and only $k$-divisors $A = \{A_\lambda, a_\lambda\}_{\lambda\in\Lambda}$ with $a_\lambda = 1/C$ const are considered. However, the lemma (page 309 and 319) and Satz 4 (page 313) yield easily this more general formulation.} *W. Stoll*