

Exercises in Algebraic Topology (master)

Prof. Dr. Birgit Richter

Summer term 2019

Exercise sheet no 8

due: 5th of June 2019

1 (Ext)

- 1) Determine $\text{Ext}(A, B)$ if A is a finitely generated abelian group.
- 2) For natural numbers n and m give an explicit formula for $\text{Ext}(\mathbb{Z}/n\mathbb{Z}, \mathbb{Z}/m\mathbb{Z})$.

2 (Products of Moore spaces) Let A and B be finitely generated abelian groups. Determine the homology groups of the product of the Moore spaces $M(A, n)$ and $M(B, m)$, $H_*(M(A, n) \times M(B, m))$, for arbitrary natural numbers $n, m \geq 1$.

In the special case of $M(\mathbb{Z}/2\mathbb{Z}, 1) = \mathbb{R}P^2$, what is $H_*(\mathbb{R}P^2 \times \mathbb{R}P^2)$?

3 (Splittings)

- a) Give an explicit example for the fact that the splitting in the topological Künneth formula is not natural.
- b) Let C_* be a free chain complex and let G be an abelian group. Show that the sequence

$$0 \rightarrow H_n(C_*) \otimes G \rightarrow H_n(C_* \otimes G) \rightarrow \text{Tor}(H_{n-1}(C_*), G) \rightarrow 0$$

splits.

4 ((Co)Homology of real projective spaces)

Calculate $H^m(\mathbb{R}P^n; \mathbb{Z}/2\mathbb{Z})$ and $H^m(\mathbb{R}P^n; \mathbb{Z})$ for all $m \geq 0$ and $n \geq 1$.