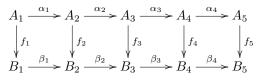
Exercises in Algebraic Topology (master)

Prof. Dr. Birgit Richter Summer term 2019

Exercise sheet no 5

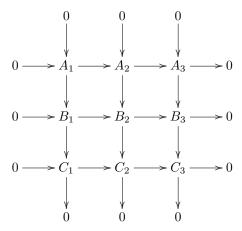
due: 15th of May 2019

1 (5-Lemma revisited) Consider the following commutative diagram of exact sequences



Check under which assumptions on f_1, f_2, f_4, f_5 we can deduce that the map f_3 is a monomorphism or an epimorphism.

2 (The $9 = 3 \times 3$ -lemma) We consider the following commutative diagram with exact columns:



- (1) Prove that the top row is exact if the two bottom rows are exact and that the bottom row is exact if the two top rows are exact.
- (2) What happens if the top and bottom rows are exact? Can you deduce that the middle row is exact or do you need an extra condition?

3 (Degrees)

- (1) Prove the Brouwer fixed-point theorem: Let X be a closed ball $B_R(x) \subset \mathbb{R}^n$ for $n \ge 1$ and let f be a continuous map $f: B_R(x) \to B_R(x)$. Prove that f has a fixed point.
- (2) Use this to show that every $(a_{ij}) = A \in M(n \times n; \mathbb{R})$ with non-negative a_{ij} must have an eigenvector with non-negative coordinates.

4 (More linear algebra) Let $A \in O(n+1)$. Then multiplication by A induces a continuous self-map on \mathbb{S}^n . What is its degree?