Exercises in Algebraic Topology (master)

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Exercise sheet no 3 due: 24th of April 2019

1 (Linear algebra)
Compare the homology groups of $GL_n(\mathbb{R})$ and $O(n)$. What about $GL_n(\mathbb{C})$ and $U(n)$?

2 (Snake Lemma)
Prove the famous Snake Lemma:
If

$$\begin{array}{ccccccc} & & A' & \xrightarrow{\alpha} & A & \xrightarrow{\beta} & A'' \longrightarrow 0 \\ & & \downarrow{\scriptstyle f'} & & \downarrow{\scriptstyle f} & & \downarrow{\scriptstyle f''} \\ 0 & \longrightarrow & B' & \xrightarrow{\alpha'} & B & \xrightarrow{\beta'} & B'' \end{array}$$

is a commutative diagram with exact rows, then there is an exact sequence

$$\ker(f') \longrightarrow \ker(f) \longrightarrow \ker(f'') \xrightarrow{\delta} \operatorname{coker}(f') \longrightarrow \operatorname{coker}(f) \longrightarrow \operatorname{coker}(f'').$$

Define δ explicitly in this case.
(For an alternative: `http://www.youtube.com/watch?v=etbcKWEKnvg`)

3 (Exactness and homomorphisms) Let M be an abelian group and let

$$0 \longrightarrow A \xrightarrow{\alpha} B \xrightarrow{\beta} C \longrightarrow 0$$

be a short exact sequence of abelian groups.
What can you say about the exactness of the sequence

$$0 \longrightarrow \operatorname{Hom}(M, A) \xrightarrow{\alpha_*} \operatorname{Hom}(M, B) \xrightarrow{\beta_*} \operatorname{Hom}(M, C) \longrightarrow 0?$$