Exercises in Algebraic Topology (master)

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Summer term 2019

Exercise sheet no 2 due: 17th of April 2019

1 (Induced maps)

a) Let X and Y be topological spaces. Is every chain map $f_*\colon S_*(X) \to S_*(Y)$ induced by a map of topological spaces?

b) Let $p\colon \tilde{X} \to X$ be a covering map. We know that the induced map on fundamental groups is a monomorphism. Is that also true for $H_1(p)$?

2 (Cones)

Let $f\colon A_* \to B_*$ be a chain map. The *mapping cone* of f, $C(f)$, is a chain complex with $C(f)_n = A_{n-1} \oplus B_n$ and whose differential is $D(a,b) = (-da, db - f(a))$. Prove that this *is* a chain complex.

Show that f_* is null-homotopic if and only if f_* extends over $C(\mathrm{id}_{A_*})$.

3 (Klein bottle and surfaces)

a) Let F_g denote the closed orientable surface of genus g. Use the Seifert van Kampen theorem to determine the fundamental group of F_g and then apply the Hurewicz theorem to calculate $H_1(F_g)$.

b) Do the same for the Klein bottle, K.

4 (Exactness)

Let C_* be an arbitrary chain complex and let p be a prime. Is it always true that the sequence of chain complexes

$$0 \longrightarrow C_* \xrightarrow{p\cdot} C_* \xrightarrow{\pi} C_*/pC_* \longrightarrow 0$$

is exact? Give a proof or a counterexample.