Exercises in Algebraic Topology (master)

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Summer term 2017

Exercise sheet no 2 for the 8th of May 2017

1 (Snake Lemma)
Deduce the famous Snake Lemma from something you learned in the lecture course:
If

$$\begin{array}{ccccccc} & & A' & \xrightarrow{\alpha} & A & \xrightarrow{\beta} & A'' \longrightarrow 0 \\ & & \downarrow{\scriptstyle f'} & & \downarrow{\scriptstyle f} & & \downarrow{\scriptstyle f''} \\ 0 & \longrightarrow & B' & \xrightarrow{\alpha'} & B & \xrightarrow{\beta'} & B'' \end{array}$$

is a commutative diagram with exact rows, then there is an exact sequence

$$\ker(f') \to \ker(f) \to \ker(f'') \xrightarrow{\delta} \operatorname{coker}(f') \to \operatorname{coker}(f) \to \operatorname{coker}(f'').$$

Define δ explicitly in this case.
(For an alternative: `http://www.youtube.com/watch?v=etbcKWEKnvg`)

2 (Right complement?)
Let $n \geqslant 0$ be any natural number. Can you find a pair of spaces (X, A) such that A is not the empty set and

$$H_0(X, A) \cong H_0(X \backslash A) \cong \mathbb{Z}^n?$$

3 (Too ugly?)
What can you say about $H_1(\mathbb{R}, \mathbb{Q})$? Is it free abelian? Does it have torsion?

4 (Linear algebra)
Compare the homology groups of $GL_n(\mathbb{R})$ and $O(n)$. What about $GL_n(\mathbb{C})$ and $U(n)$?