

Exercises in Algebraic Topology (master)

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Summer term 2015

Exercise sheet no 8

For the exercise class on the 3rd of June 2015

29 (Right exactness)

- a) Prove the right exactness of the tensor product.
- b) Is the abelian group \mathbb{Q} free? Is the functor $A \mapsto A \otimes \mathbb{Q}$ exact?

30 (The discrete circle as a boomerang?)

Let A be a finitely generated abelian torsion group. Can you identify $\text{Hom}(A, \mathbb{Q}/\mathbb{Z})$ and/or $\text{Tor}(A, \mathbb{Q}/\mathbb{Z})$ with A ?

31 (How bad can it be?)

Give an example of a chain complex (C_*, d) with trivial homology, such that the chain complex $C_* \otimes \mathbb{Z}/2\mathbb{Z}$ has non-vanishing homology in every degree.

32 (Same for R -modules?)

Assume that R is a commutative ring with unit. Can you define Tor for R -modules in the same way as we did for $R = \mathbb{Z}$? What is different?

What happens if R is a field? What about $R = \mathbb{Z}/4\mathbb{Z}$?